Turing Machine II

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Adapted From:
IALC 8.2,8.3.1,8.3.2,8.4.1,8.4.2,8.6.1,8.6.2,9.2

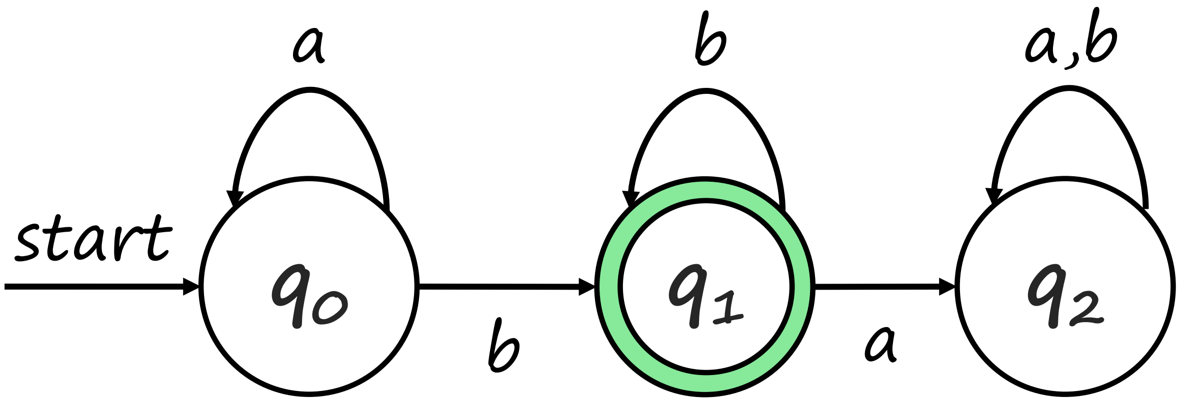
Review

- **Function**
  - Definition and common functions.
  - Injective, surjective, bijective.
  - Inverse of function.
- **Computers are complex.**
- **Automata is a mathematical model of a computing device**
  - Clean and simple
  - DFA: Five tuples $(Q, \Sigma, \delta, q_0, F)$

Review: Deterministic Finite Automaton(DFA)

- A DFA takes a finite string(sequence of input symbols) as input, and process the input symbols of the string one by one, and change its state according to the transition function δ.
- DFA will always stop, if the state is in the set of accepting states(F) when a DFA stops, we say that this DFA **accepts** the input. Otherwise it **rejects** the input.

Simpler Notations for DFA



Transition Diagram

start state

	a	b
$\rightarrow q_0$	q_0	q_1
$* q_1$	q_2	q_1
q_2	q_2	q_2

final state

Transition Table

Extending the Transition Function

For a DFA $(Q, \Sigma, \delta, q_0, F)$, we can define an ***extended transition function***

$$\hat{\delta}: Q \times \Sigma^* \rightarrow Q$$

$\hat{\delta}$ takes a state q and a string w as input, and returns the state after processing string w from state q.

- $\hat{\delta}(q, \epsilon) = q$
- If $w = xa$, where $a \in \Sigma$. Then $\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$

Extending the Transition Function

- $\hat{\delta}(q_0, \epsilon) = q_0$
- $\hat{\delta}(q_0, 1) = \delta\big(\hat{\delta}(q_0, \epsilon), 1\big) = \delta(q_0, 1) = q_1$
- $\hat{\delta}(q_0, 11) = \delta\big(\hat{\delta}(q_0, 1), 1\big) = \delta(q_1, 1) = q_0$
- $\hat{\delta}(q_0, 110) = \delta\big(\hat{\delta}(q_0, 11), 0\big) = \delta(q_0, 0) = q_2$
- $\hat{\delta}(q_0, 1101) = \delta\big(\hat{\delta}(q_0, 110), 1\big) = \delta(q_2, 1) = q_3$
- $\hat{\delta}(q_0, 11010) = \delta\big(\hat{\delta}(q_0, 1101), 0\big) = \delta(q_3, 0) = q_1$
- $\hat{\delta}(q_0, 110101) = \delta\big(\hat{\delta}(q_0, 11010), 1\big) = \delta(q_1, 1) = q_0$

	0	1
$* \rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

The Language of DFA

- For a DFA $\boldsymbol{A} = (Q, \Sigma, \delta, q_0, F)$, the language of $\boldsymbol{A}$, donated as $L(\boldsymbol{A})$, is the set of all string that A accepts.

$$L(A) = \{w | \hat{\delta}(q_0, w) \in F\}$$

- For a certain language L, if there exists a DFA $\boldsymbol{A}$ such that $L = L(\boldsymbol{A})$, then we call L is a **regular language(正则语言)**.

We'll not take much time in DFA and regular language. Feel free to read chapter 2,3,4 in IALC if you are interested in it.

Regular Expression(正则表达式)

- Regular expressions are used to denote regular languages.
- For an alphabet Σ
  - $\emptyset, \epsilon, a$ are regular expressions corresponding to languages $\emptyset, \{\epsilon\}, \{a\}$, where $a \in \Sigma$
  - If $\boldsymbol{r}, \boldsymbol{s}$ are regular expression corresponding to language L_r, L_s.
    - $\boldsymbol{r} + \boldsymbol{s}$ (or $\boldsymbol{r}|\boldsymbol{s}$): $L_r \cup L_s$
    - $\boldsymbol{rs}$: $\{wu | w \in L_r \wedge u \in L_s\}$
    - $\boldsymbol{r}^{\boldsymbol{k}}$: $\{w_1 w_2 \cdots w_{k-1} w_k | w_1 \in L_r \wedge w_2 \in L_r \wedge \cdots w_k \in L_r\}$
    - $\boldsymbol{r}^+$: $L_{r^1} \cup L_{r^2} \cup L_{r^3} \ldots \cup L_{r^n} \ldots$
    - $\boldsymbol{r}^*$: $L_{r^+} \cup \{\epsilon\}$

Regular Expression(正则表达式)

- Recognizing phone numbers using RegEx

```
(([0-9]{3}-)?[0-9]{8})
```

Three digits

Pattern XXX- occurs once or never

Capture (XXX-)XXXXXXXX

Regular Expression(正则表达式)

- Use RegEx in C++: std::regex

```cpp
#include <iostream>
#include <iterator>
#include <string>
#include <regex>

int main()
{
    std::string s = "自2012年5月至今，由上海交通大学保卫处打造的“54749110”报警求助平台，"
                    "在校内已然成为一张“交大名片”。该报警热线不仅有效提供校内警情处置、应急强险、帮困求助、询问"
                    "答疑等多样化服务，而且更是党委机关密切联系师生、贴近人民群众的有效途径。保卫处以“全心全意"
                    "为师生服务”为成立初衷，在校内事务处置等多个方面充分践行着“决不让任何一名交大人在需要帮助的"
                    "时候感到困惑和无助”的服务承诺，获得了师生的广泛好评，被校内师生美誉为“万能的9110”。2019"
                    "年上线至今，021-54741234生活服务平台在学校校园管理与服务委员会的指导下，以生活服务“总客"
                    "服”为角色定位汇聚师生生活密切相关的13大类81小类公共服务，串联17职能单位与学院，一站式为师"
                    "生 “办实事“以 “解难题“。 ";

    std::regex phone_regex("(([0-9]{3}-)?[0-9]{8})");
    auto phones_begin =
        std::sregex_iterator(s.begin(), s.end(), phone_regex);
    auto phones_end = std::sregex_iterator();

    std::cout << "找到"
              << std::distance(phones_begin, phones_end)
              << "个电话号码\n";

    for (std::sregex_iterator i = phones_begin; i != phones_end; ++i)
    {
        std::smatch match = *i;
        std::string match_str = match.str();
        if (match_str.size() > 3)
        {
            std::cout << "  " << match_str << '\n';
        }
    }

    std::regex long_word_regex("(\\w{7,})");
    std::string new_s = std::regex_replace(s, long_word_regex, "[$&]");
    std::cout << new_s << '\n';
}
```

- Include RegEx library
  - `#include <regex>`
- Declare pattern `(([0-9]{3}-)?[0-9]{8})`
  - `std::regex phone_regex("(([0-9]{3}-)?[0-9]{8})");`
- Use regex iterators on target string
  - `std::sregex_iterator`

Regular Expression(正则表达式)

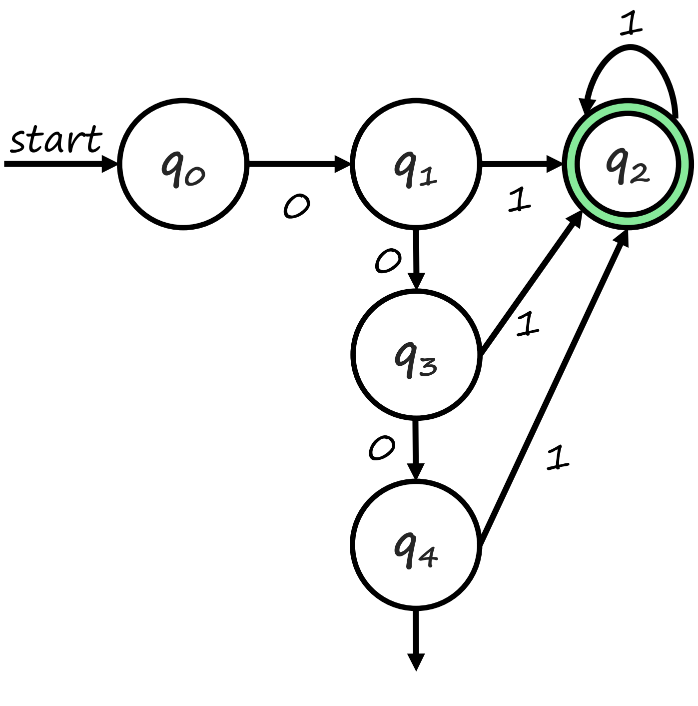
- **Use RegEx in C++: std::regex**

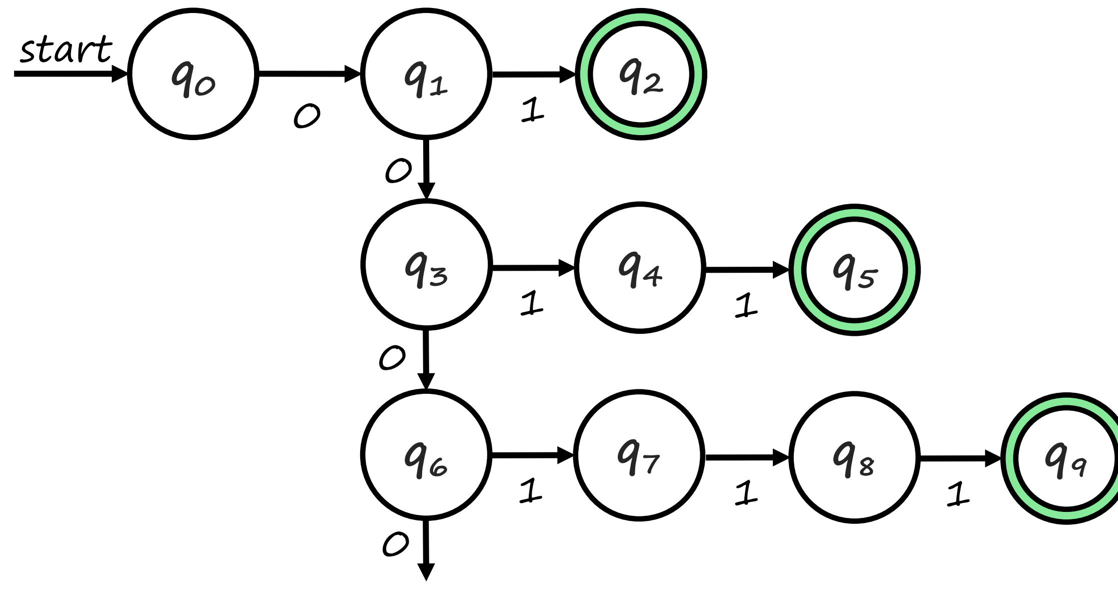
```
找到2个电话号码
  54749110
  021-54741234
自2012年5月至今，由上海交通大学保卫处打造的“[54749110]”报警求助平台，在校内已然成为一张“交大名片”。该报警热线不仅有效提供校内警情处置、应急强险、帮困求助、询问答疑等多样化服务，而且更是党委机关密切联系师生、贴近人民群众的有效途径。保卫处以“全心全意为师生服务”为成立初衷，在校内事务处置等多个方面充分践行着“决不让任何一名交大人在需要帮助的时候感到困惑和无助”的服务承诺，获得了师生的广泛好评，被校内师生美誉为“万能的9110”。2019年上线至今，021-[54741234]生活服务平台在学校校园管理与服务委员会的指导下，以生活服务“总客服”为角色定位汇聚师生生活密切相关的13大类81小类公共服务，串联17职能单位与学院，一站式为师生 “办实事“以 “解难题“。
```

But DFA can not recognize all the languages.

The Limitation of DFA

- There is no DFA for languages like $\{0^n1^n | n \in \mathbb{N}^+\}$.



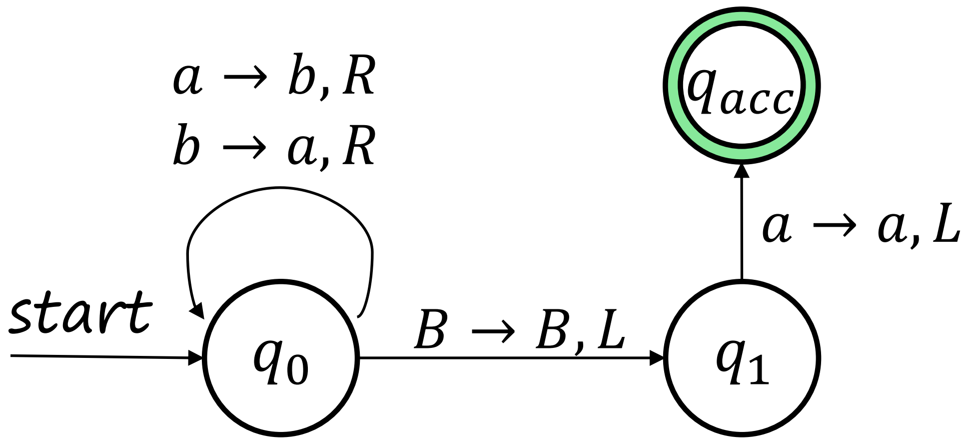


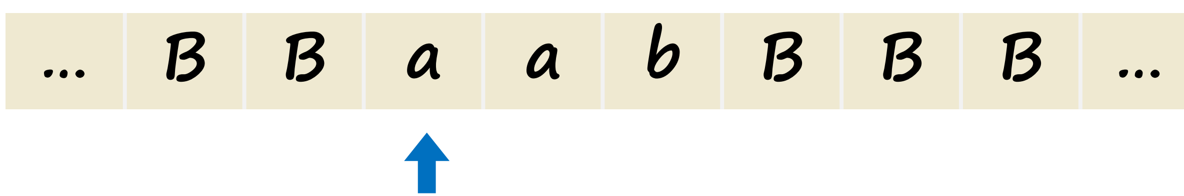
The Limitation of DFA

- There is no DFA for languages like $\{0^n1^n | n \in \mathbb{N}^+\}$.
- We need a more powerful “machine” to model our computer.

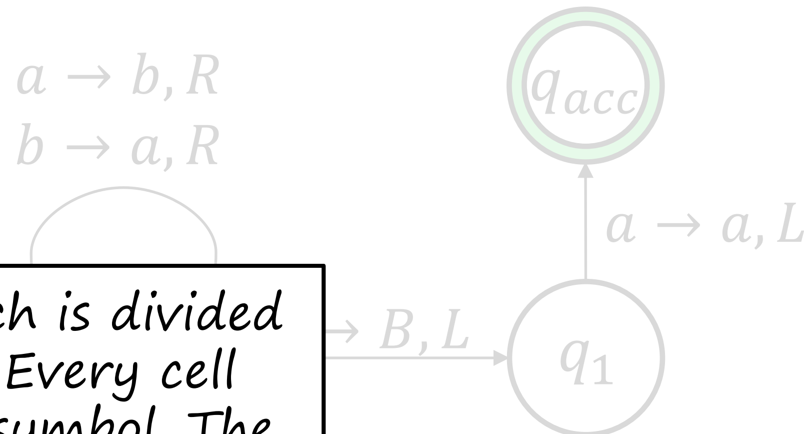
Turning Machine!!

Turing Machine





Turing Machine

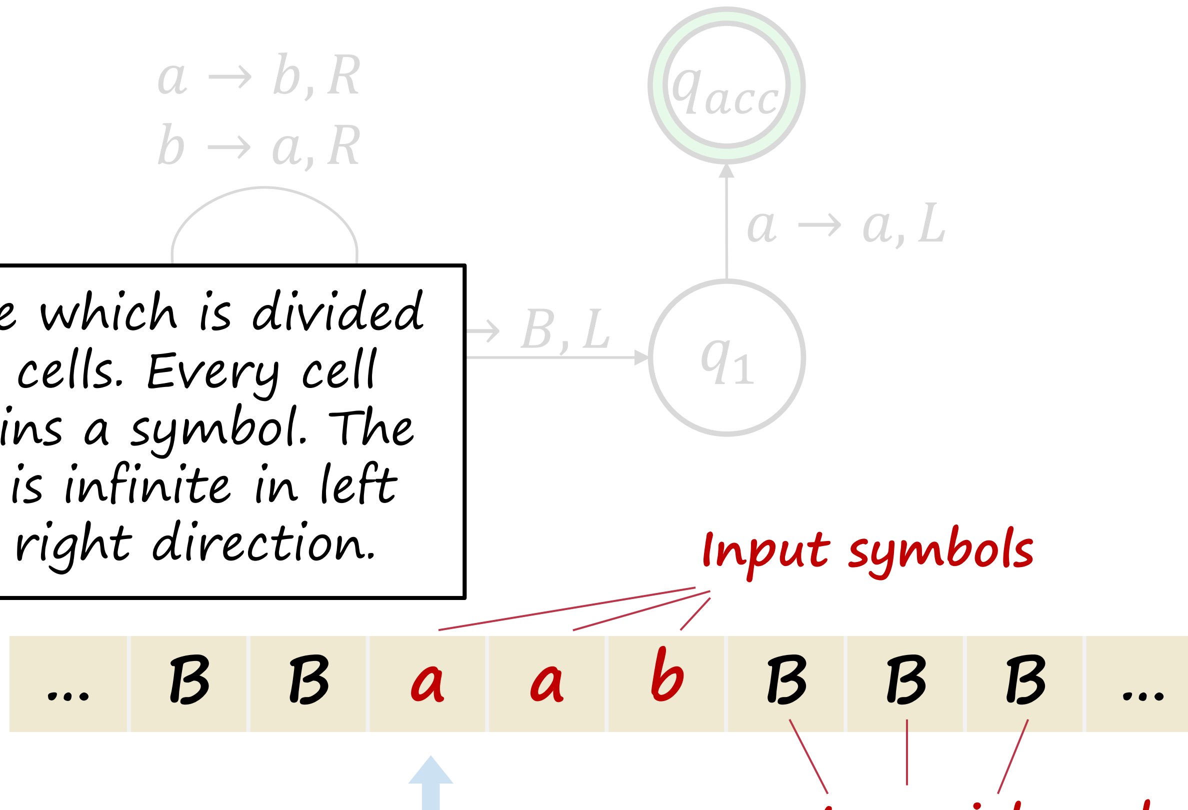


A tape which is divided into cells. Every cell contains a symbol. The tape is infinite in left and right direction.

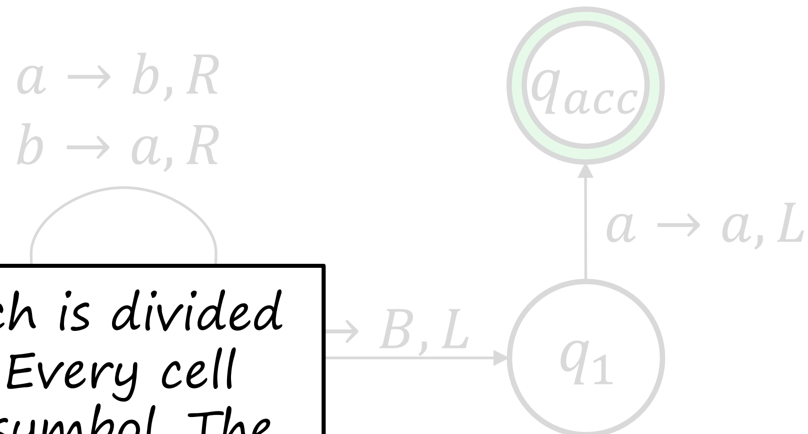
…	B	B	a	a	b	B	B	B	…

Turing Machine

A tape which is divided into cells. Every cell contains a symbol. The tape is infinite in left and right direction.



Turing Machine

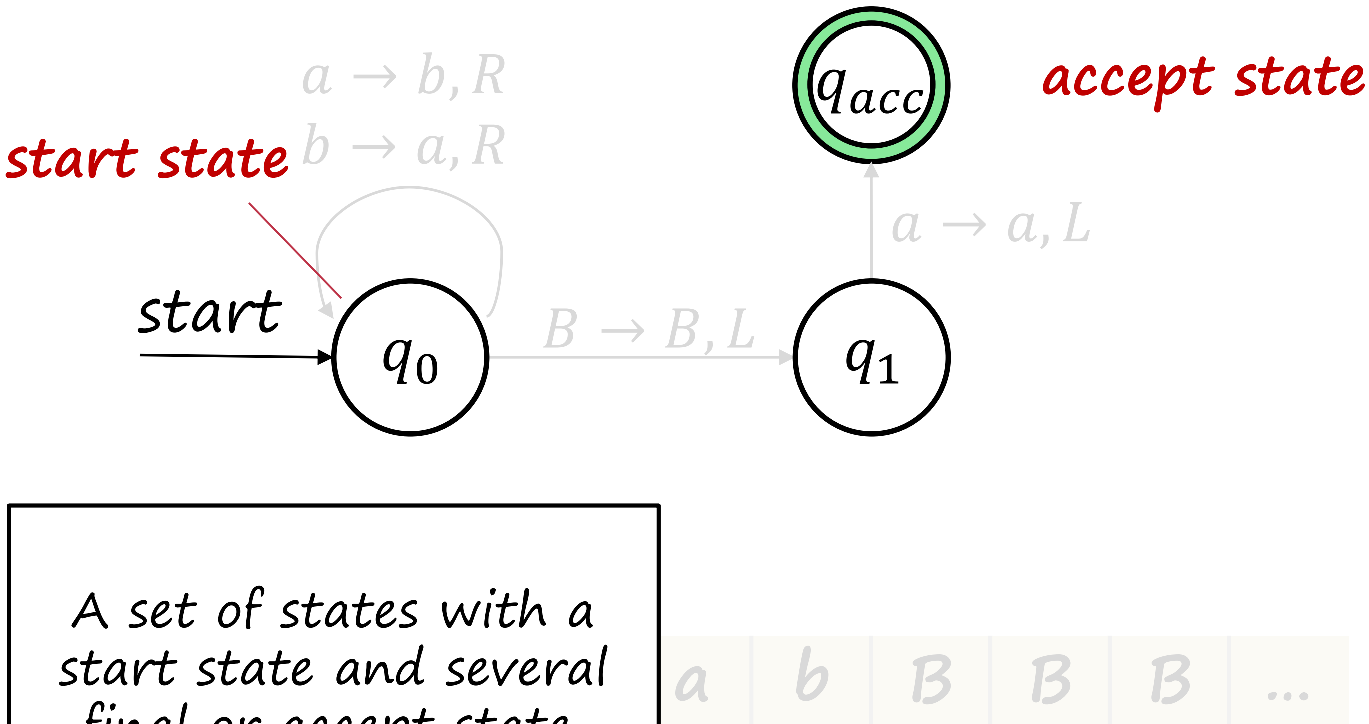


A tape which is divided into cells. Every cell contains a symbol. The tape is infinite in left and right direction.

…	B	B	a	a	b	B	B	B	…

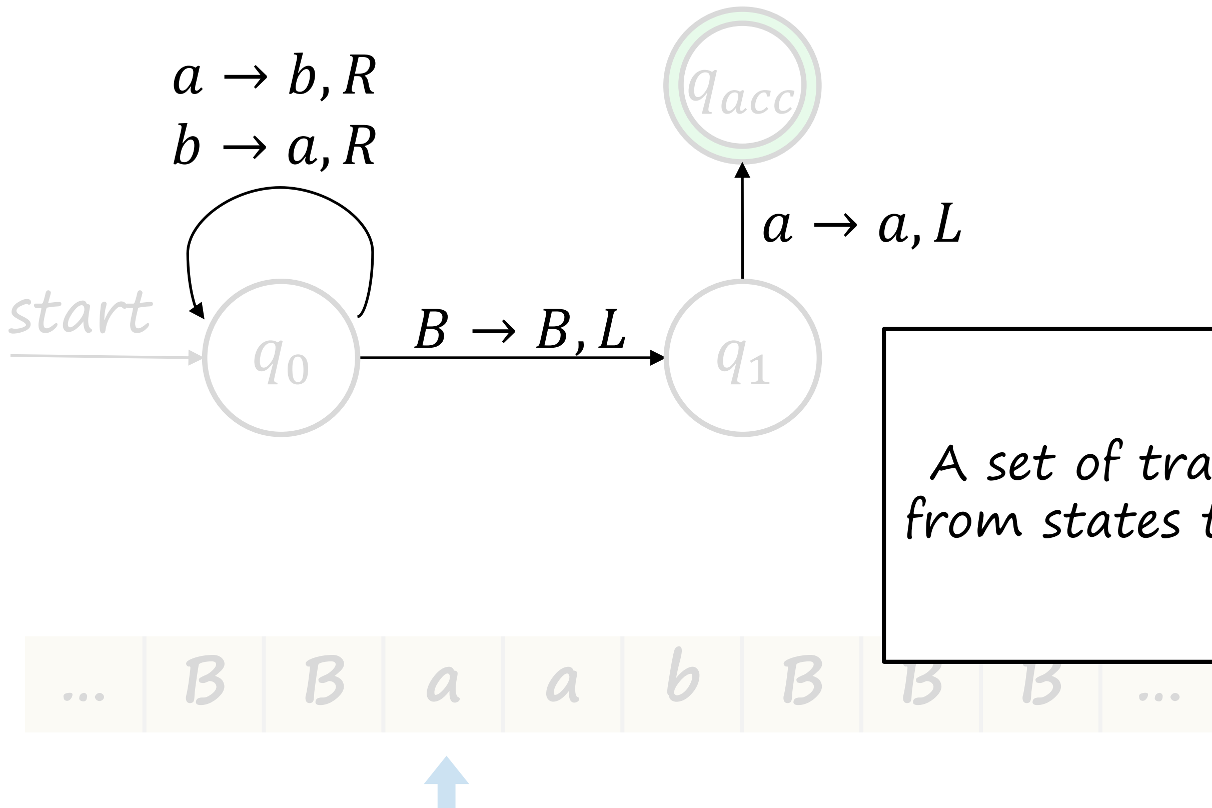
A tape head which initially points to the start of input symbol

Turing Machine



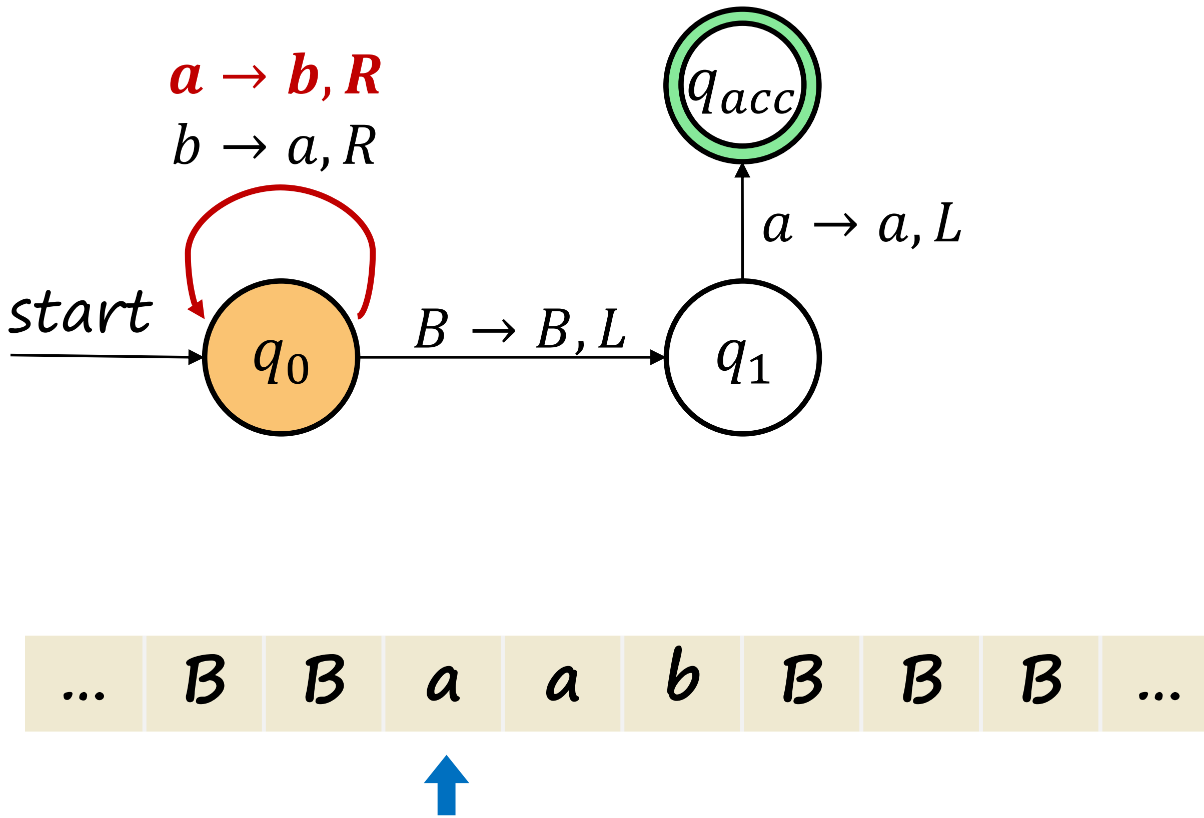
A set of states with a start state and several final or accept state.

Turing Machine

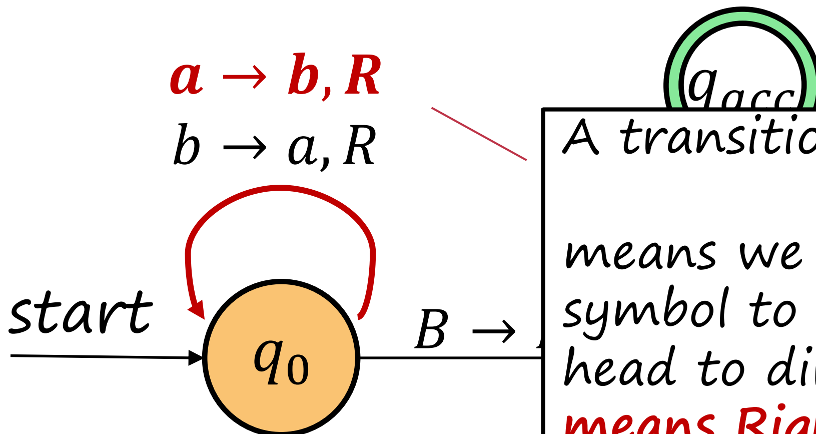


A set of transitions from states to states.

Turing Machine



Turing Machine



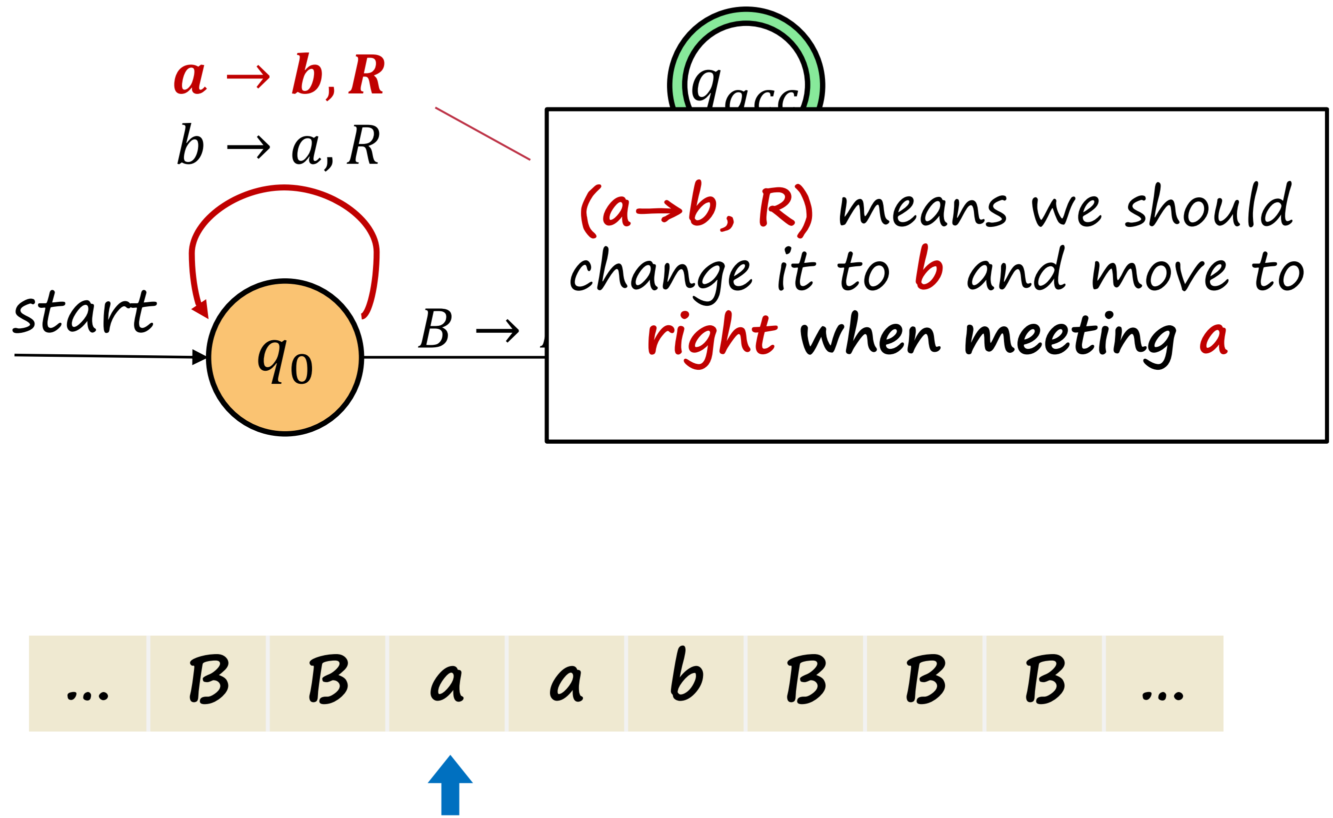
A transition has the form:

(s1→s2, d)

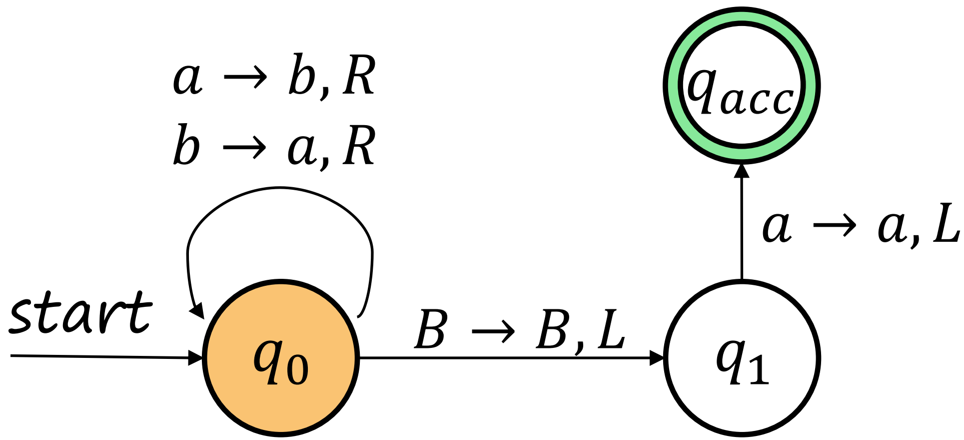
means we should change the symbol to **s2** and move the tape head to direction **d (d∈{R,L}, means Right or Left)** when meeting **s1**.

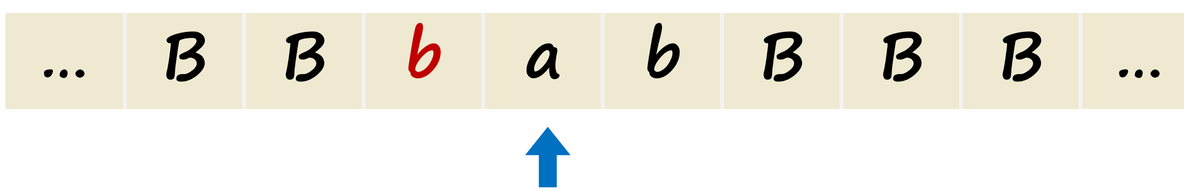
…	B	B	a	a	b	B	B	B	…

Turing Machine

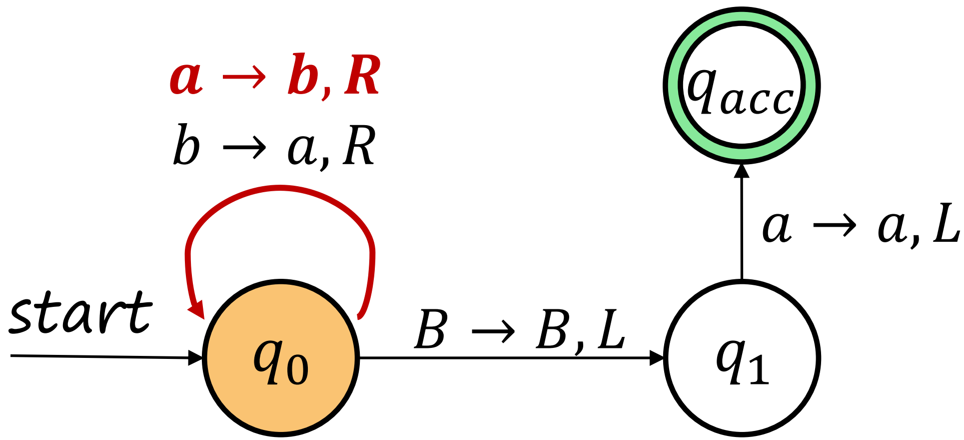


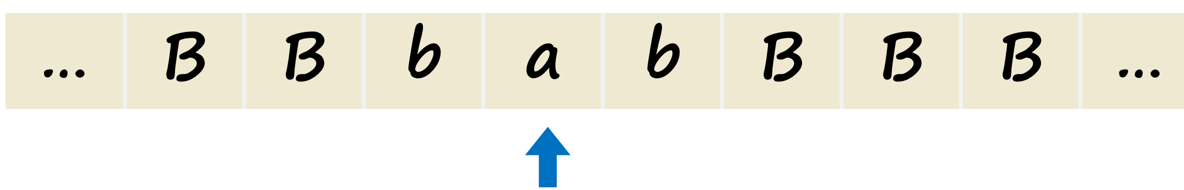
Turing Machine



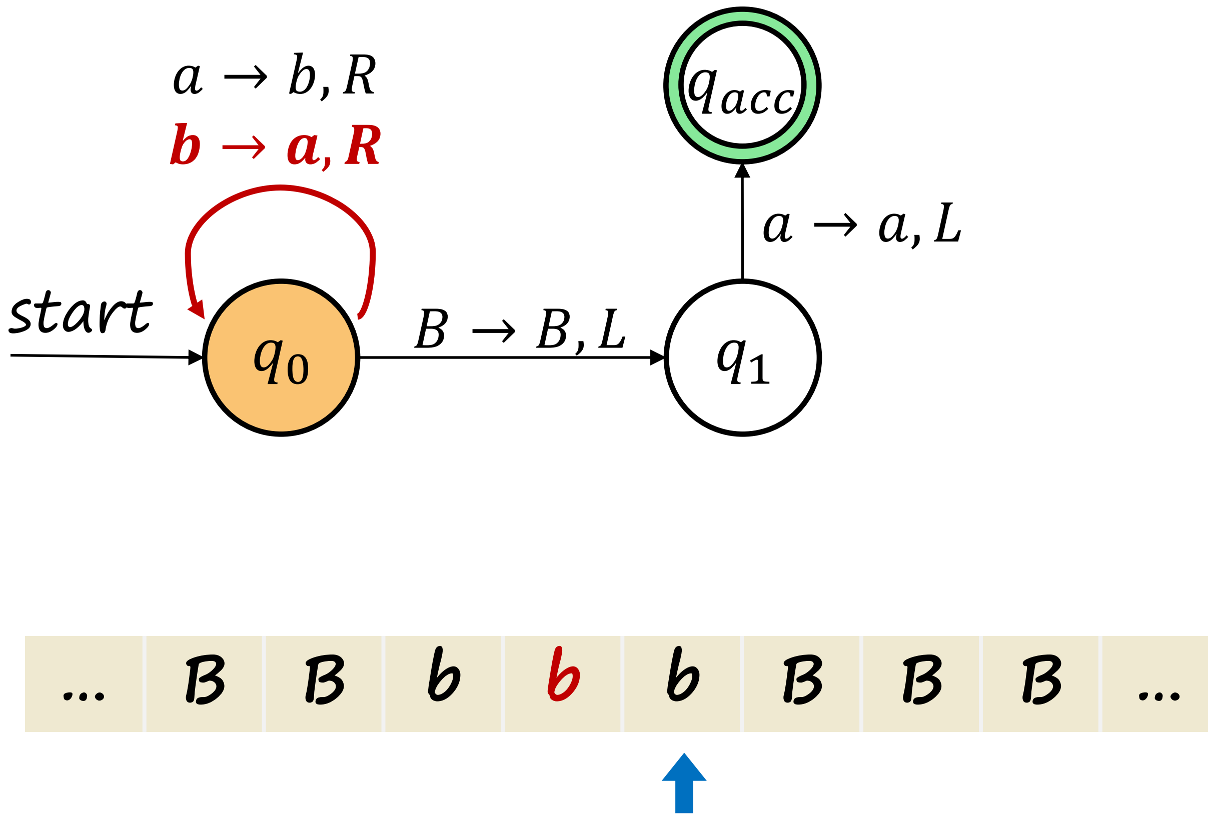


Turing Machine

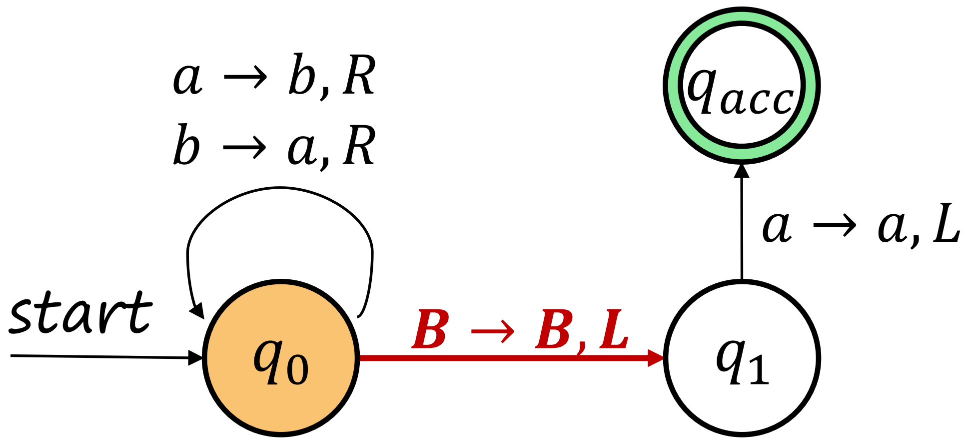


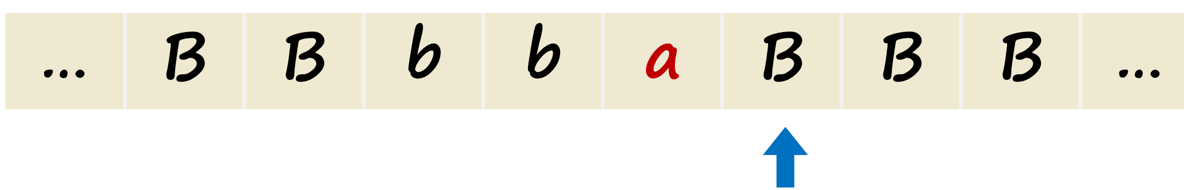


Turing Machine

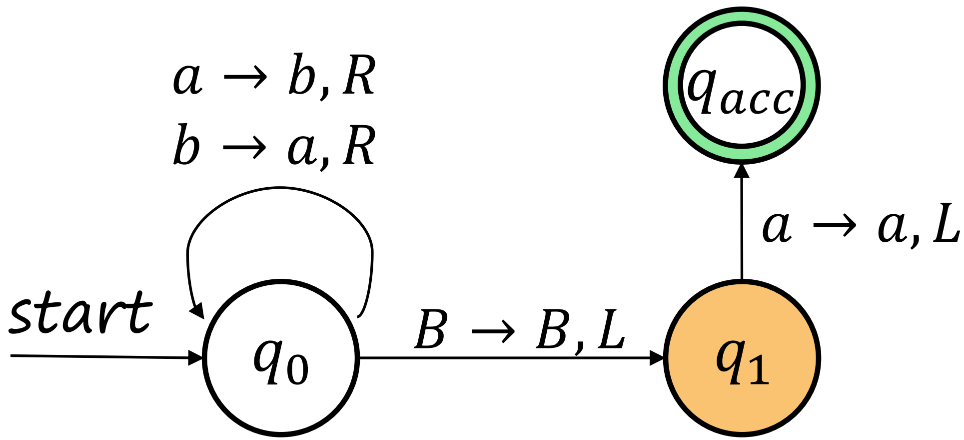


Turing Machine

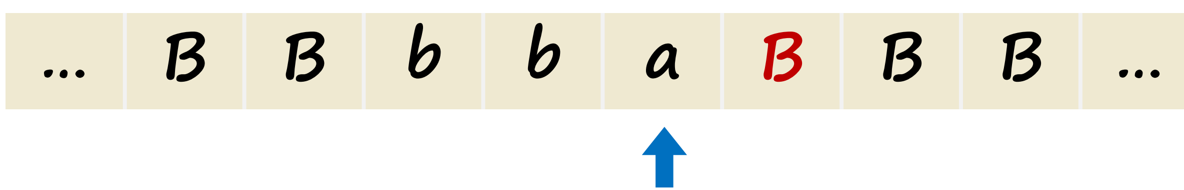




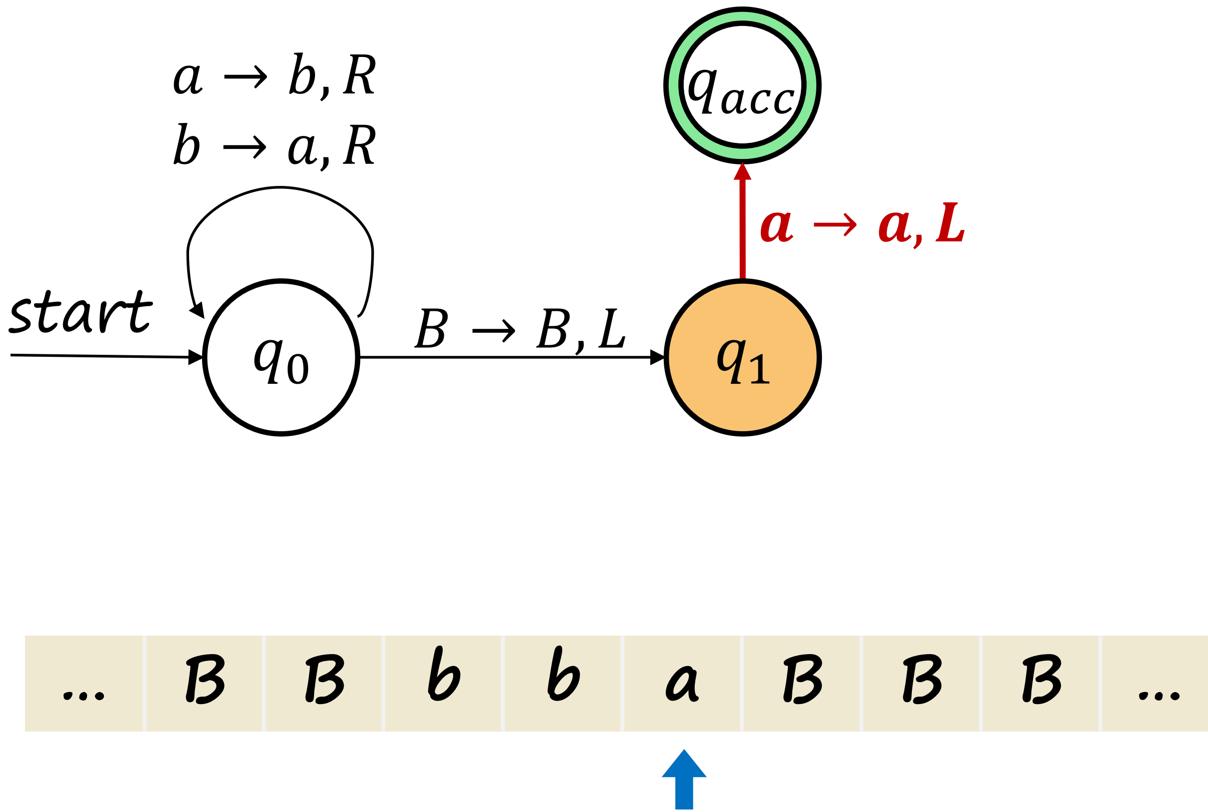
Turing Machine



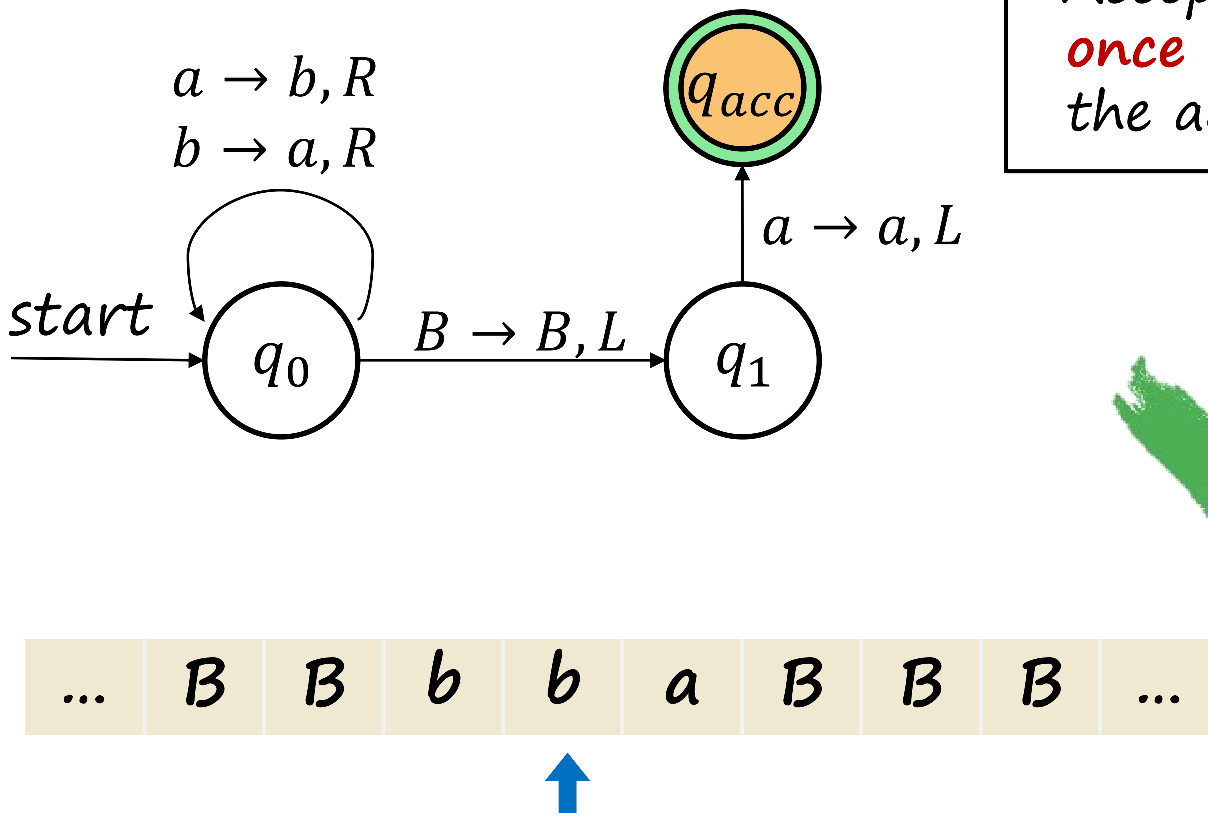
It's ok not to change the tape.



Turing Machine

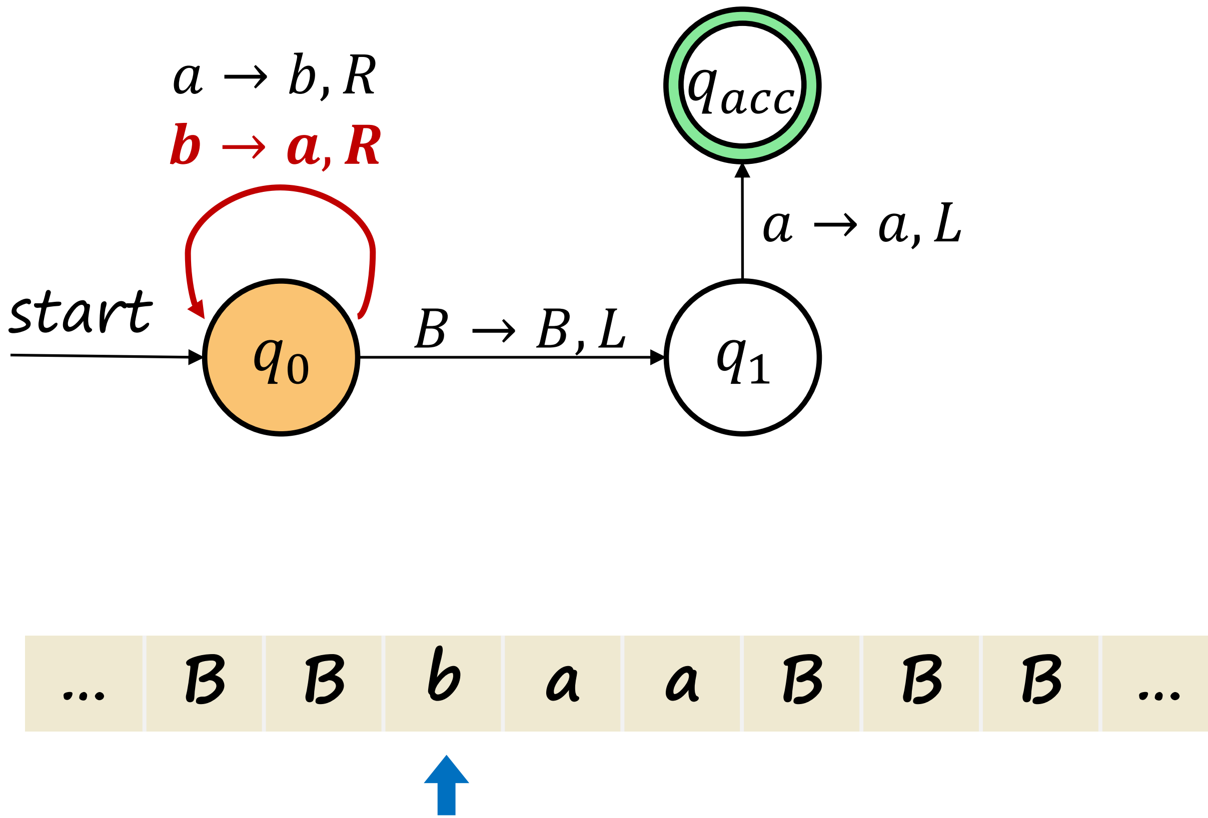


Turing Machine

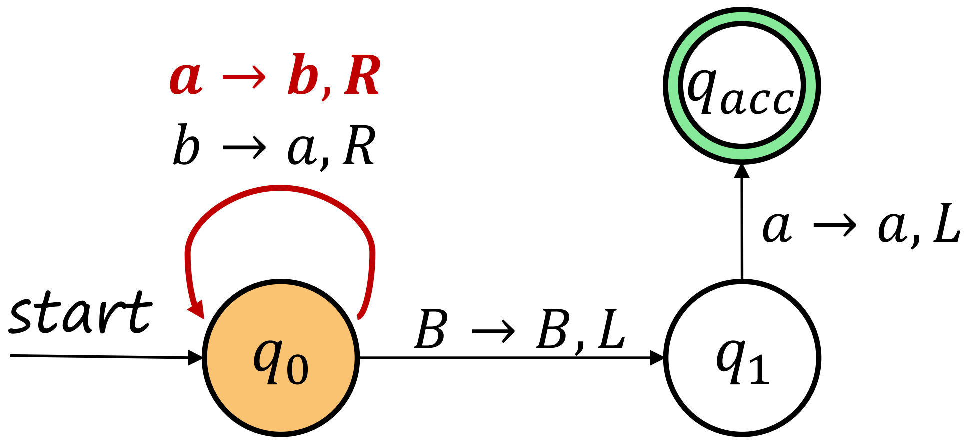


Accept the input **once** we get into the accept state.

Turing Machine

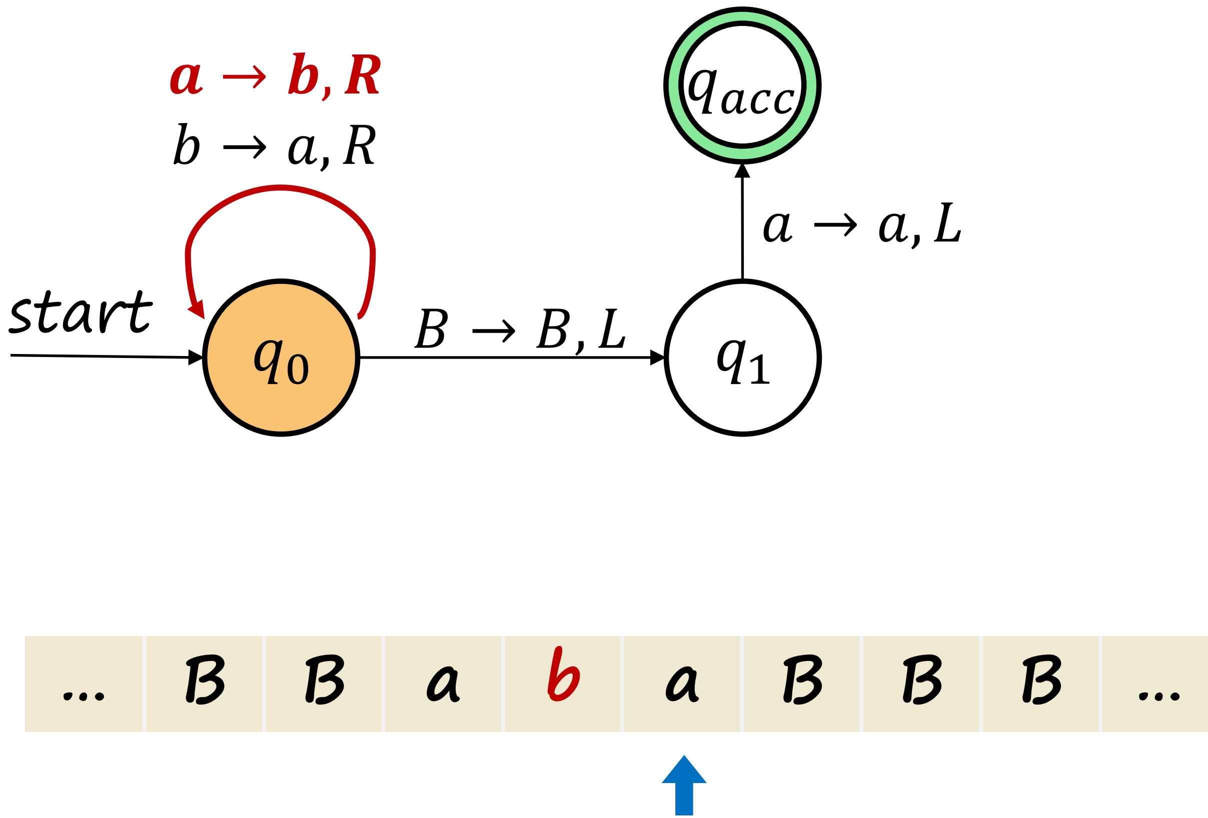


Turing Machine

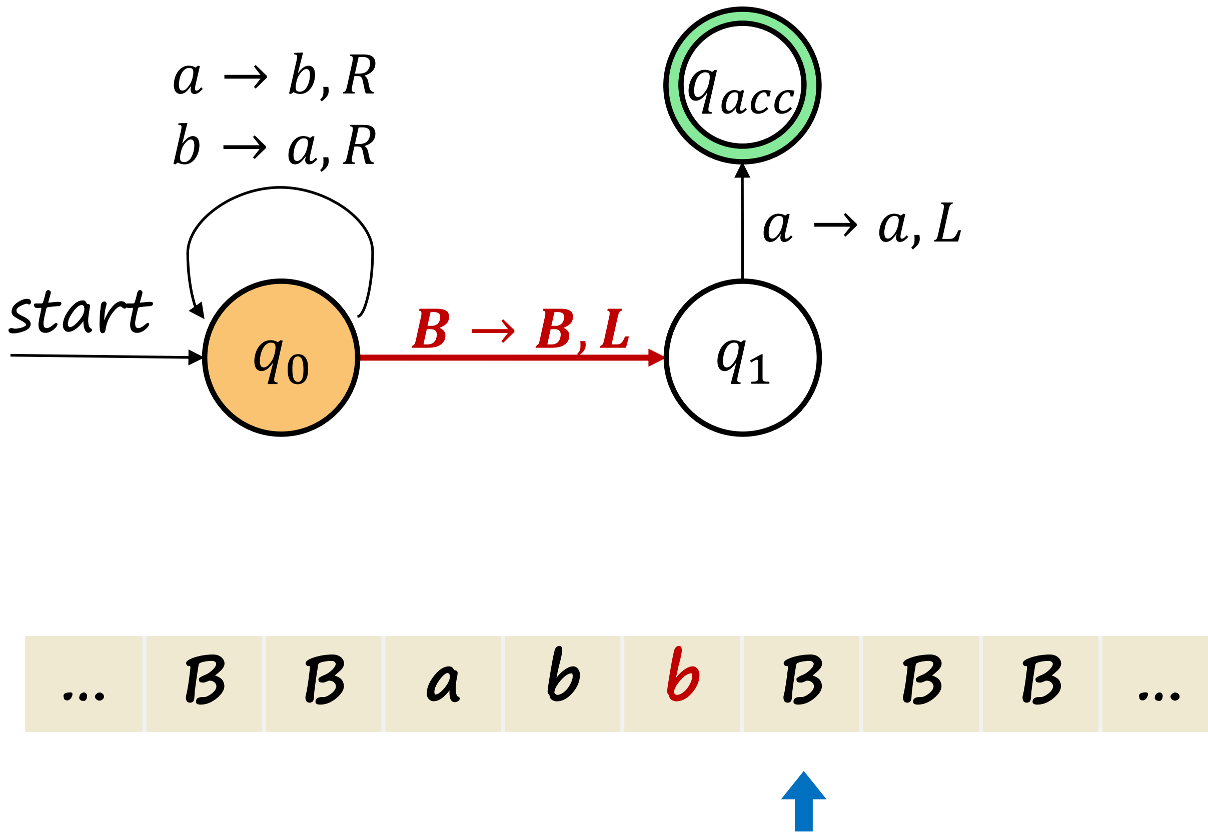


…	B	B	a	a	a	B	B	B	…

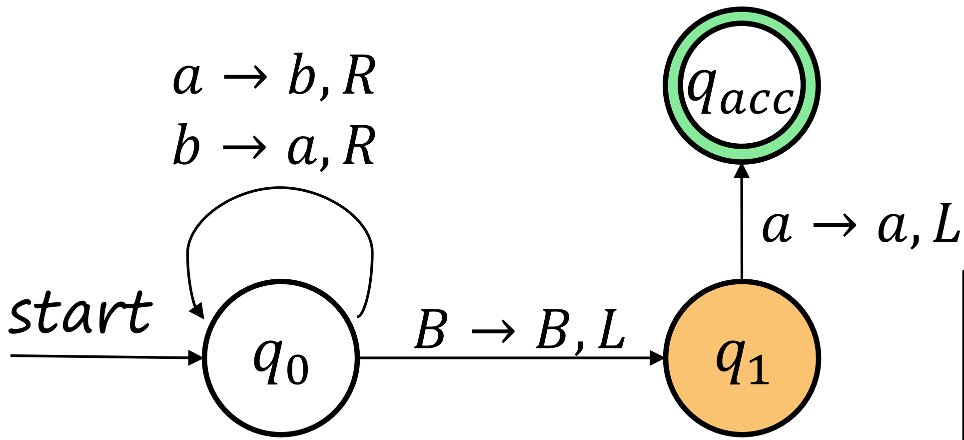
Turing Machine



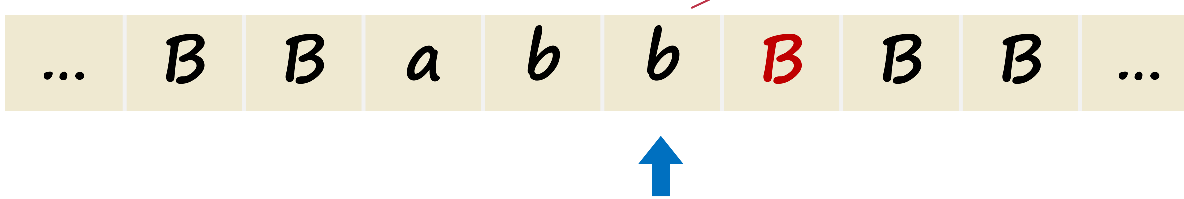
Turing Machine



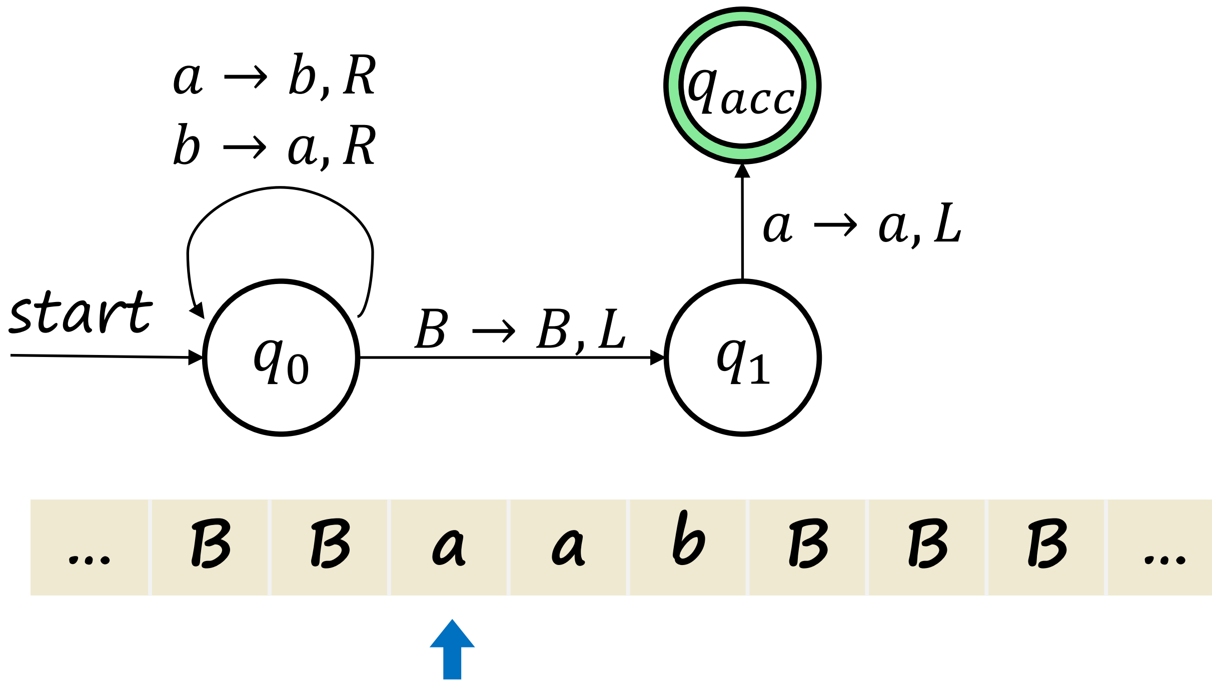
Turing Machine



We have no direction to go. The computation finishes, but doesn't accept the input



Turing Machine



{w | w∈{a,b}*∧ (w ends with b)}

Turing Machine

- A **Turing Machine** consists of:
  - Q:A finite set of states
  - Σ: A finite set of input symbols
  - Γ: The complete set of tape symbols. $\Sigma \subset \Gamma$
  - δ: A transition function. $Q \times \Sigma \rightarrow Q \times \Gamma \times \{L, R\}$
  - q_0: The start state. $q_0 \in Q$
  - B: The blank symbol. $B \in \Gamma \wedge B \notin \Sigma$.
  - F: A set of final or accepting states. $F \subseteq Q$
- We can donate a turning machine as a "seven tuple"

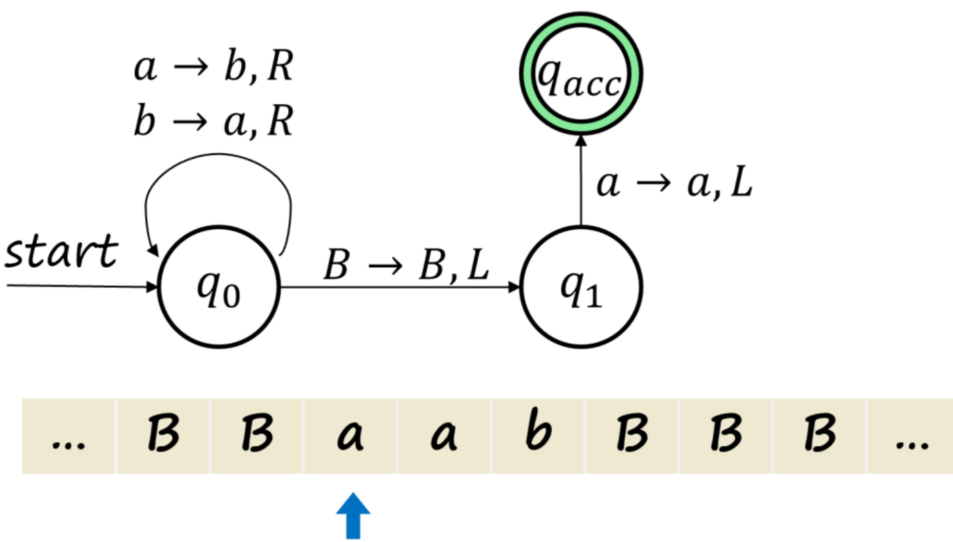
$$A = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

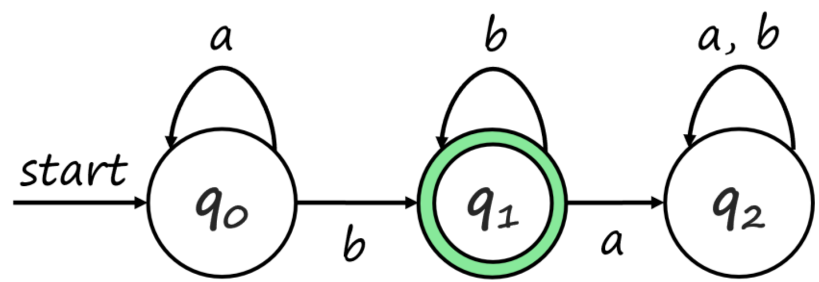
Turing Machine

- Initially the input symbols are put into tape, and the tape heap is pointed to the start of input symbols.
- Turing Machine finishes computation by moving the tape head. In a move, the Turing Machine will:
  - Change state
  - Write a tape symbol in the cell pointed
  - Move the tape head left or right.
- Turing Machine will accept the input once it gets into accept states, or finishes computation with rejection when no move can be made.

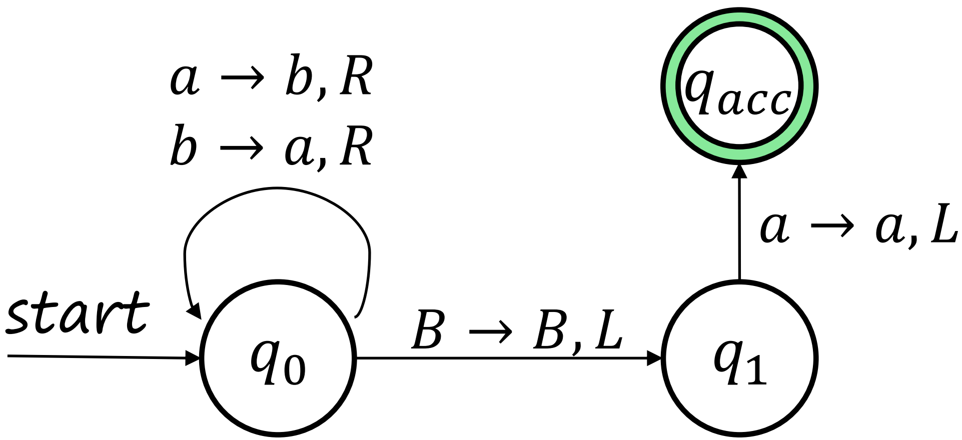
TM and DFA

- DFA has only a bunch of states but TM has an extra "memory-like" tape to store the execution state.
- TM has more capability than DFA
  - A DFA can be converted into a TM





Simpler Notations for TM



Transition Diagram

start state

final state

	a	b	B
$\to q_0$	(q_0, b, R)	(q_0, a, R)	(q_1, B, L)
q_1	(q_{acc}, a, L)	$-$	$-$
$* \; q_{acc}$	$-$	$-$	$-$

Transition Table

Example: TM for $\{0^n1^n\}$

- Count the number of 0 and the number of 1, and compare them.
  - But we have no idea how to store the number and compare them.
- Or when we see a 0, we try to find an 1, and repeat the process until we count every symbol.

Example: TM for $\{\mathbf{0}^n\mathbf{1}^n\}$

- Our solution:
  - When we see a $\mathbf{0}$ at the beginning, we change this $\mathbf{0}$ to $\boldsymbol{X}$ to indicate we have counted it, and try to find an $\mathbf{1}$ by moving tape head to right.
  - When we find an $\mathbf{1}$, we change this $\mathbf{1}$ to $\boldsymbol{Y}$, and turn back to find another $\mathbf{0}$ at the beginning (right after an $\boldsymbol{X}$).
  - Repeat the above process when there are only $\boldsymbol{X}$ and $\boldsymbol{Y}$ left on the tape, then we accept the input. Otherwise we reject the input.

Example: TM for $\{0^n 1^n\}$

We've found a 0

...	B	0	0	0	1	1	1	B	...

Example: TM for $\{0^n1^n\}$

Change it to X

…	B	X	0	0	1	1	1	B	…

Example: TM for $\{0^n 1^n\}$

Move right to find an 1

…	B	X	0	0	1	1	1	B	…

Example: TM for $\{0^n1^n\}$

We've found an 1

...	B	X	0	0	1	1	1	B	...

Example: TM for $\{0^n 1^n\}$

Change it to Y

...	B	X	0	0	Y	1	1	B	...

Example: TM for $\{0^n 1^n\}$

Turn back to find 0 at the beginning

…	B	X	0	0	Y	1	1	B	…

Example: TM for $\{0^n1^n\}$

Stop move left when we meet X.

…	B	X	0	0	Y	1	1	B	…

Example: TM for $\{0^n1^n\}$

Take a right move. We found another 0.

...	B	X	0	0	Y	1	1	B	...

Example: TM for $\{0^n1^n\}$

Change it to X and continue.

…	B	X	X	0	Y	1	1	B	…

Example: TM for $\{0^n1^n\}$

Another 1.

...	B	X	X	0	Y	1	1	B	...

Example: TM for $\{0^n1^n\}$

Change it to Y and turn back again.

...	B	X	X	0	Y	Y	1	B	...

Example: TM for $\{0^n 1^n\}$

Meet X, stop moving left.

…	B	X	X	0	Y	Y	1	B	…

Example: TM for $\{0^n1^n\}$

Another 0.

…	B	X	X	0	Y	Y	1	B	…

Example: TM for $\{0^n 1^n\}$

Change it to X.

...	B	X	X	X	Y	Y	1	B	...

Example: TM for $\{0^n 1^n\}$

Another 1.

...	B	X	X	X	Y	Y	1	B	...

Example: TM for $\{0^n 1^n\}$

Change it and turn back again.

…	B	X	X	X	Y	Y	Y	B	…

Example: TM for $\{0^n1^n\}$

Meet X, take a right move.

...	B	X	X	X	Y	Y	Y	B	...

Example: TM for $\{0^n 1^n\}$

There's no 0 left. We need to check if there's only X and Y left on the tape.

...	B	X	X	X	Y	Y	Y	B	...

Example: TM for $\{0^n1^n\}$

We only need to check towards the right direction as we know the left are all X.

…	B	X	X	X	Y	Y	Y	B	…

Example: TM for $\{0^n1^n\}$

We stop and accept the result until we meet a B.

…	B	X	X	X	Y	Y	Y	B	…

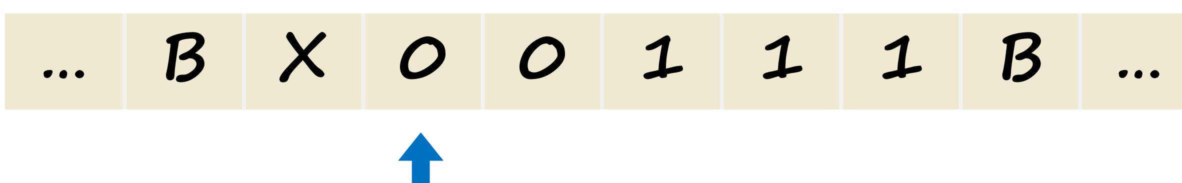
Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$					

...	B	0	0	0	1	1	1	B	...

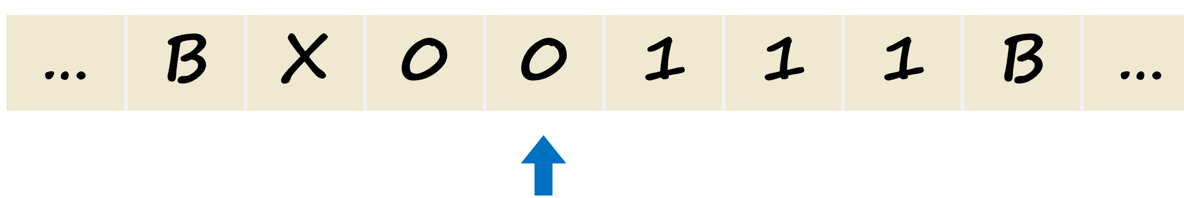
Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1					



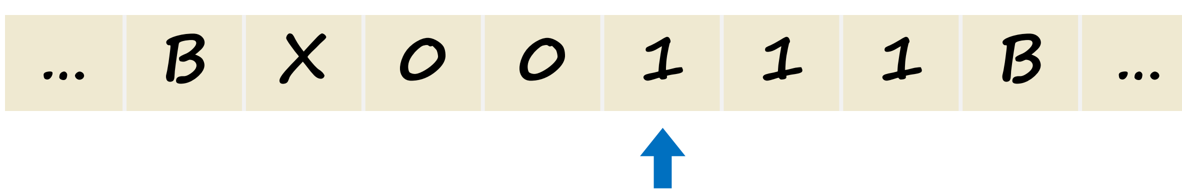
Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$				



Example: TM for $\{0^n1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$				



Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)			
q_2					

…	B	X	0	0	Y	1	1	B	…

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)			
q_2	$(q_2, 0, L)$				

…	B	X	0	0	Y	1	1	B	…

Example: TM for $\{0^n1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)			
q_2	$(q_2, 0, L)$		$(?, X, R)$		

...	B	X	0	0	Y	1	1	B	...

Example: TM for $\{0^n1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)			
q_2	$(q_2, 0, L)$		(q_0, X, R)		

…	B	X	0	0	Y	1	1	B	…

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)			
q_2	$(q_2, 0, L)$		(q_0, X, R)		

…	B	X	X	0	Y	1	1	B	…

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)		

…	B	X	X	0	Y	1	1	B	…

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)		

...	B	X	X	0	Y	Y	1	B	...

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	

…	B	X	X	0	Y	Y	1	B	…

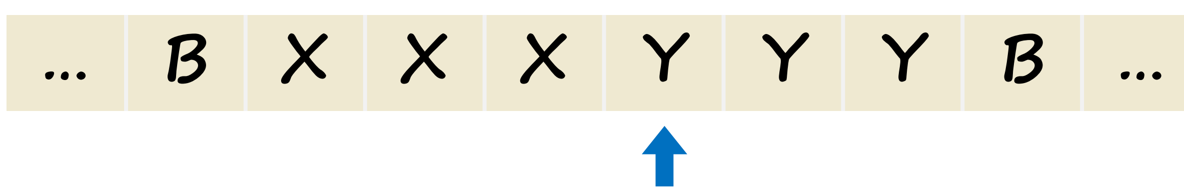
Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	

…	B	X	X	X	Y	Y	1	B	…

Example: TM for $\{0^n1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)				
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	



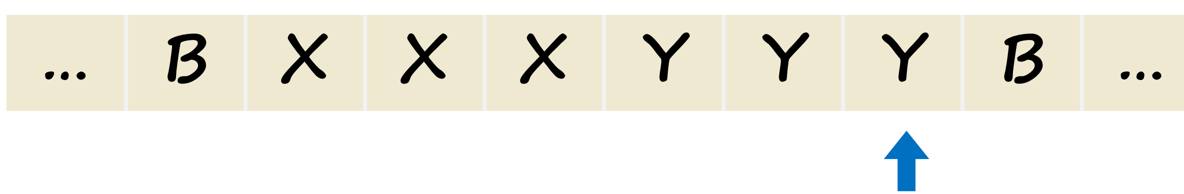
Example: TM for $\{0^n1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)			(q_3, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3					

…	B	X	X	X	Y	Y	Y	B	…

Example: TM for $\{0^n 1^n\}$

	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)			(q_3, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	



Example: TM for $\{0^n1^n\}$

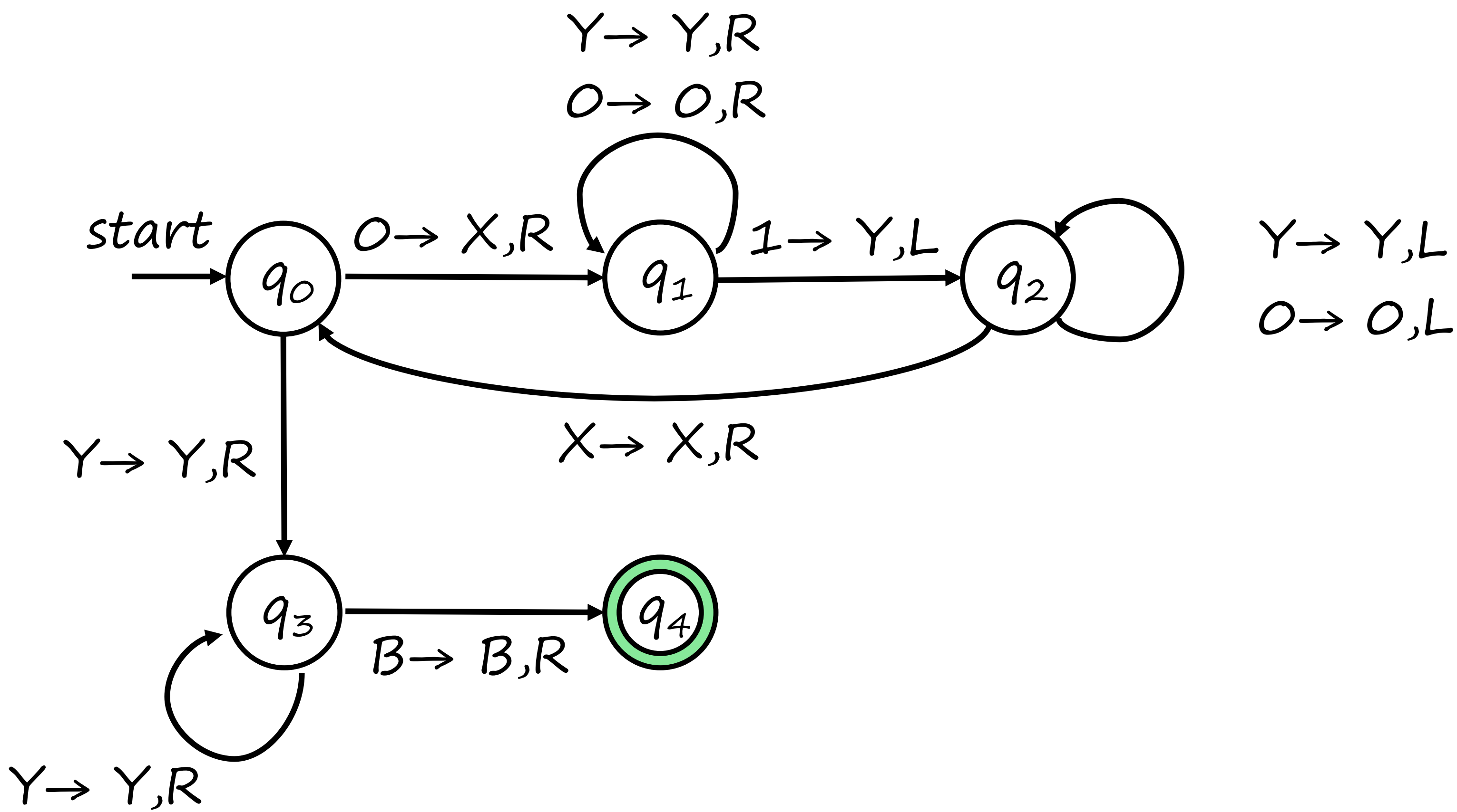
	0	**1**	**X**	**Y**	**B**
$\rightarrow q_0$	(q_1, X, R)			(q_3, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, R)
$* \ q_4$					

…	B	X	X	X	Y	Y	Y	B	…

Example: TM for $\{0^n 1^n\}$

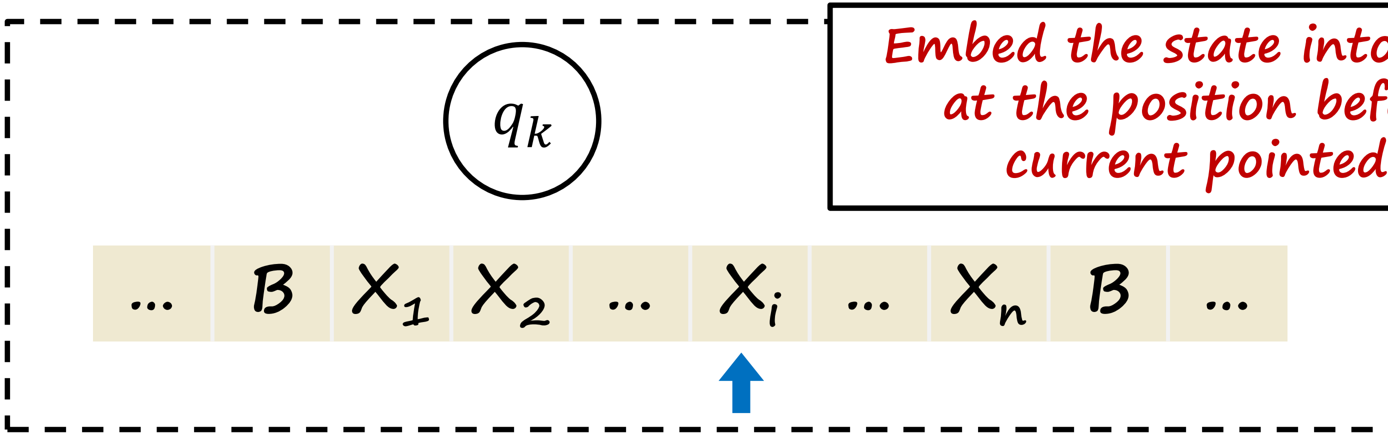
	0	1	X	Y	B
$\rightarrow q_0$	(q_1, X, R)	$-$	$-$	(q_3, Y, R)	$-$
q_1	$(q_1, 0, R)$	(q_2, Y, L)	$-$	(q_1, Y, R)	$-$
q_2	$(q_2, 0, L)$	$-$	(q_0, X, R)	(q_2, Y, L)	$-$
q_3	$-$	$-$	$-$	(q_3, Y, R)	(q_4, B, R)
$* \; q_4$	$-$	$-$	$-$	$-$	$-$

Example: TM for $\{0^n1^n\}$



Instantaneous Descriptions for TM

- **Instantaneous Descriptions (ID's)** formally describe what a Turing machine does.
  - Tape contents, Current state, Location of type head



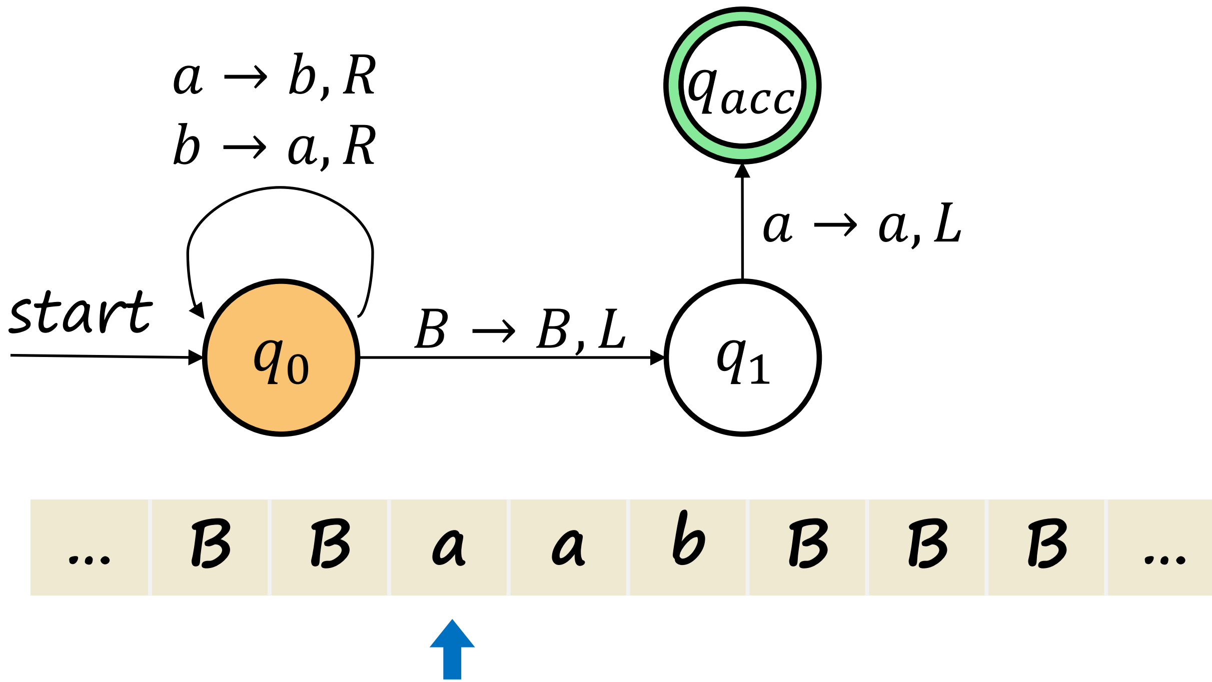
Embed the state into the tape at the position before the current pointed cell.

$$X_1X_2\ldots X_{i-1}q_kX_i\ldots X_n$$

Instantaneous Descriptions(ID's) for TM

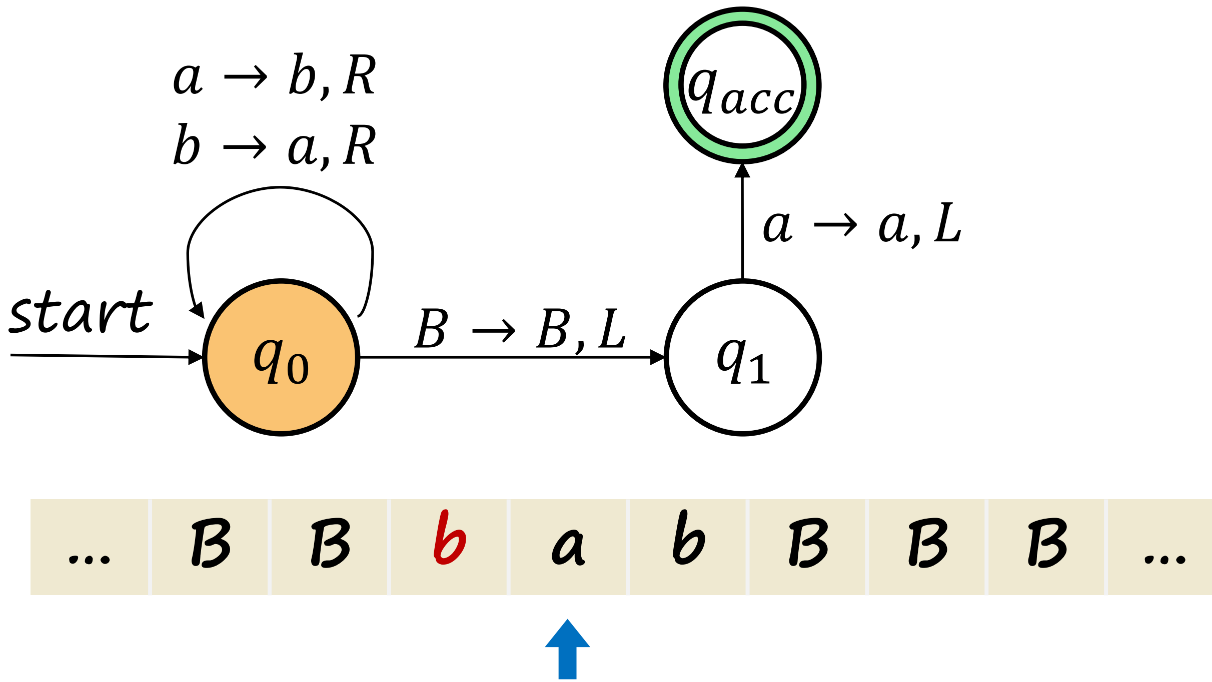
- Describe the moves of TM as $\vdash_M$
- If $\delta(q, X_i) = (p, Y, R)$ then
  - $X_1X_2 \dots X_{i-1}\boldsymbol{qX_i}X_{i+1} \dots X_n \ \vdash_M \ X_1X_2 \dots X_{i-1}\boldsymbol{Yp}X_{i+1} \dots X_n$
- If $\delta(q, X_i) = (p, Y, L)$ then
  - $X_1X_2 \dots X_{i-1}\boldsymbol{qX_i}X_{i+1} \dots X_n \ \vdash_M \ X_1X_2 \dots \boldsymbol{p}X_{i-1}\boldsymbol{Y}X_{i+1} \dots X_n$
- $\vdash_M^*$ indicated zero, one or more moves
  - $ID_1 \vdash_M^* ID_2$ means that ID_1 can be transformed to ID_2 through zero, one or more moves.

ID's and Moves



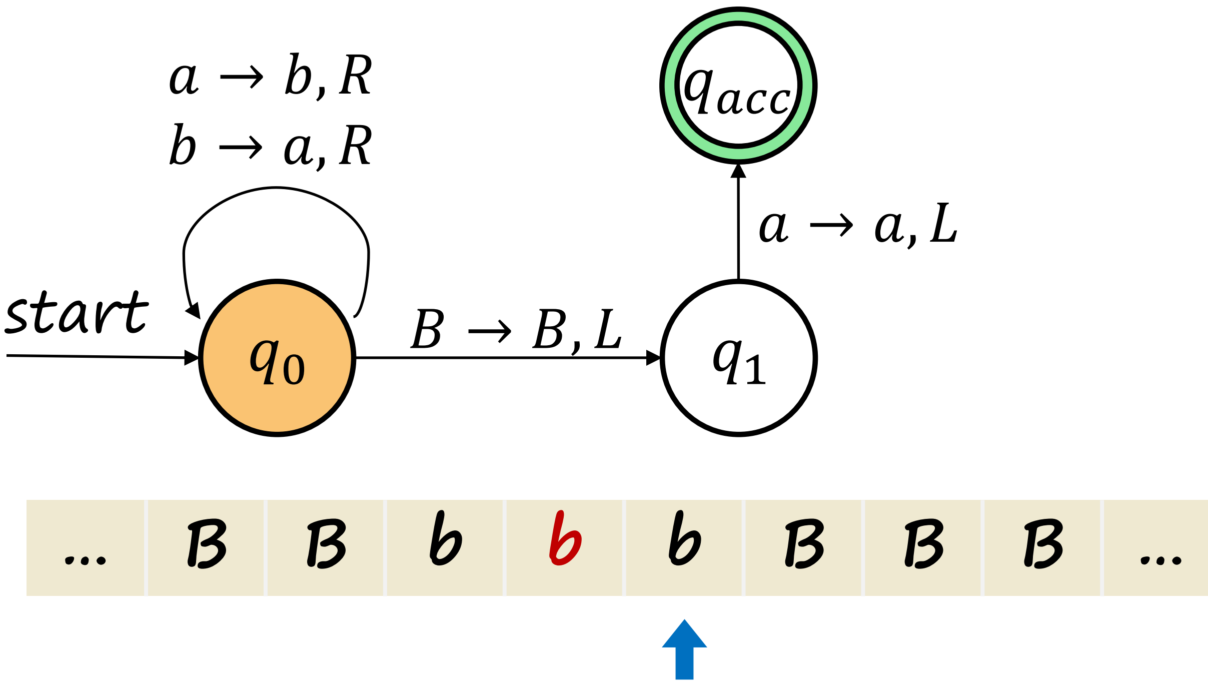
$q_0 aab$

ID's and Moves



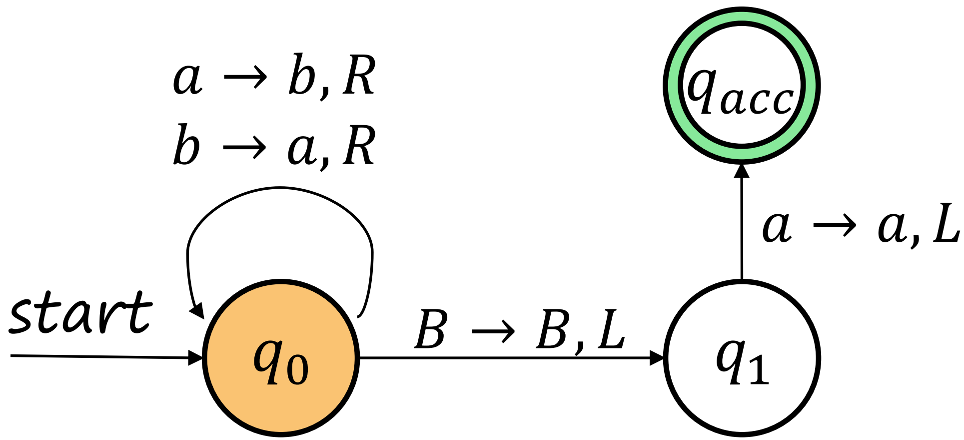
$$q_0aab \vdash_M bq_0ab$$

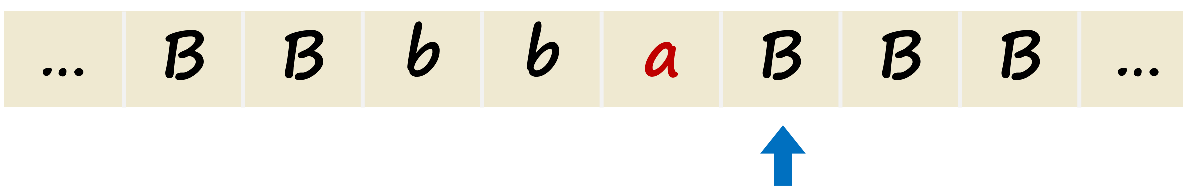
ID's and Moves



$$q_0aab \vdash_M bq_0ab \vdash_M bbq_0b$$

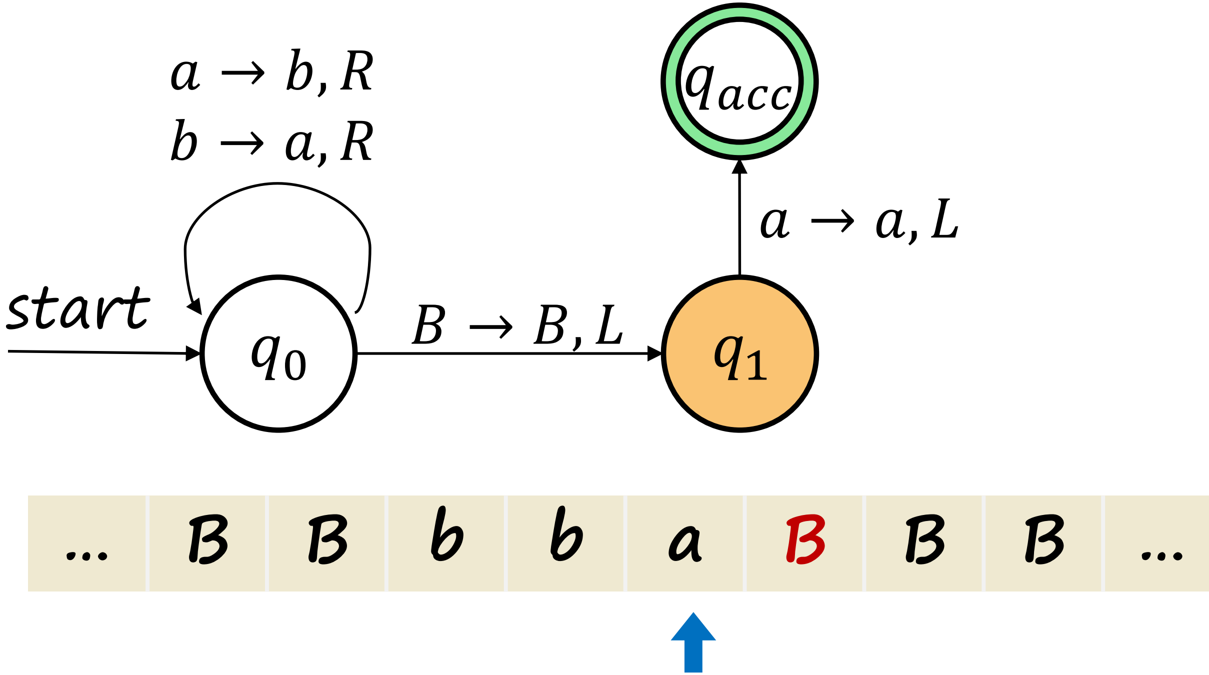
ID’s and Moves





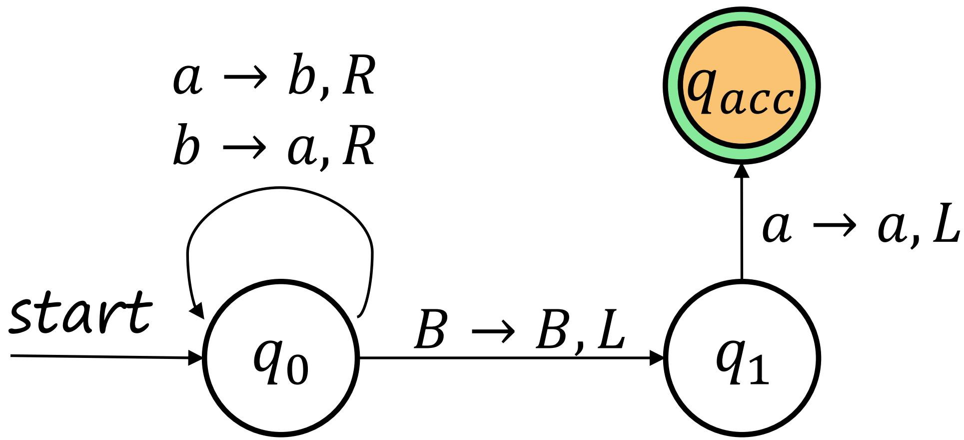
$q_0aab \vdash_M bq_0ab \vdash_M bbq_0b \vdash_M$
$bbaq_0B$

ID's and Moves



$q_0aab \vdash_M bq_0ab \vdash_M bbq_0b \vdash_M$
$bbaq_0B \vdash_M bbq_1aB$

ID's and Moves



...	B	B	b	b	a	B	B	B	...

$$q_0aab \vdash_M bq_0ab \vdash_M bbq_0b \vdash_M$$
$$bbaq_0B \vdash_M bbq_1aB \vdash_M bq_{acc}ba$$

The Language of a TM

- Similar to DFA, the language of a TM $\boldsymbol{M} = (\boldsymbol{Q}, \boldsymbol{\Sigma}, \boldsymbol{\Gamma}, \boldsymbol{\delta}, \boldsymbol{q_0}, \boldsymbol{B}, \boldsymbol{F})$ is set of strings that it accepts, or:

$$L(\boldsymbol{M}) = \{w | w \in \Sigma^* \wedge p \in F \wedge q_0 w \vdash^* \alpha p \beta\}$$

- For a certain language L, if there exists a TM $\boldsymbol{M}$ such that $L = L(\boldsymbol{M})$, then we call L is a **recursively enumerable language(递归可枚举语言)**，or ***RE***.

Computation Capability

- For any certain type of automata (e.g. DFA,TM), the computation capability of it can be regarded as all the languages the type of automata can accept.

$$P(T) = \{L(A) | A \ is \ a \ T\},$$

where T can be DFA , Turing Machine, etc.

- For DFA, the capability is all the regular languages. For Turing Machine, it is all the *RE* languages.
  - $P(DFA) = \{L | L \ is \ a \ Regular \ Language\}$
  - $P(TM) = \{L | L \ is \ a \ RE \ Language\}$

Computation Capability

- For two types of automata T_1 and T_2
  - If $P(T_1) = P(T_2)$, we say T_1 and T_2 have same capability.
  - If $P(T_1) \subset P(T_2)$, we say T_2 have greater capability than T_1.
- DFA and TM:
  - For any language $\boldsymbol{L}$ accepted by a DFA ($\boldsymbol{L} \in \boldsymbol{P}(\boldsymbol{DFA})$), there's a TM $\boldsymbol{M}$ such that $L(\boldsymbol{M}) = \boldsymbol{L}$ ($\boldsymbol{L} \in \boldsymbol{P}(\boldsymbol{TM})$), so it means that $\boldsymbol{P}(\boldsymbol{DFA}) \subseteq \boldsymbol{P}(\boldsymbol{TM})$
  - And we know there's some languages can be accepted by TM but can not be accepted by DFA(like $\{1^n 2^n\}$), so we have $P(DFA) \subset P(TM)$, which means **TM have greater computation capability than DFA**

Church-Turing Thesis

Every effective method of computation is either equivalent to or weaker than a Turing machine.

- This is not a **theorem** but a falsifiable scientific hypo**thesis**. But it has been thoroughly tested! So we have strong faith in its correctness.
- This means Turing Machine can model any “computation”.

~~What problems can a computer solve?~~

↓

~~What languages can a computer recognize?~~

↓

What languages can TM recognize?

How we investigate computability?

2.1 Discussion on Problems

Part1. Intro & Set theory(I) : Basics & Formal Language.

Part2. Set Theory(II): Axiom system & Cardinality.

Part3. Capture Structures: Binary Relation & Function

2.2 Discussion on Computation

Part1. Turing Machine Basics.

Part2. Variants of Turing Machine. Church-Turing Thesis.

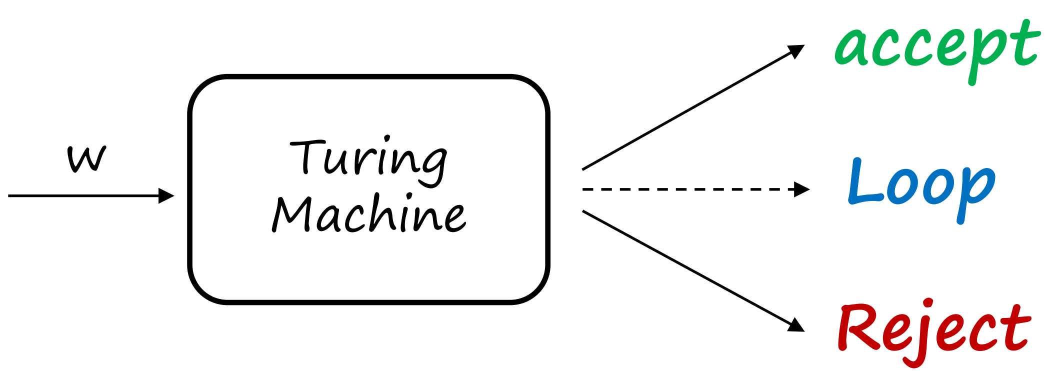
2.3 Discussion on computability

Part1. The Language of Turing Machine. R & RE

Part2. Undecidability.

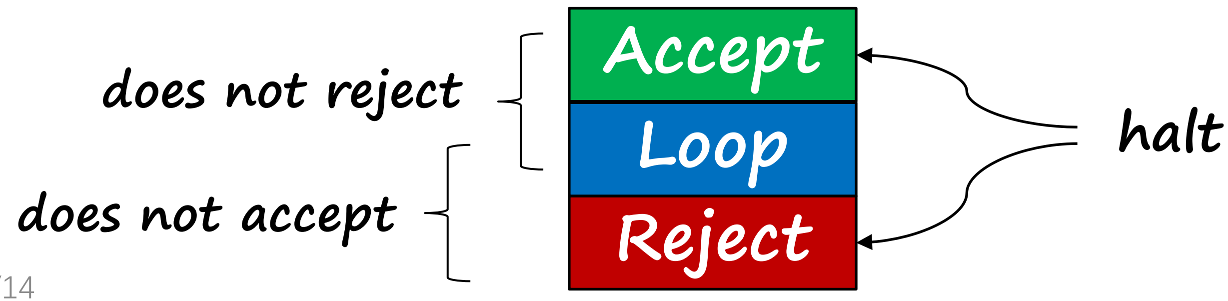
Output of TM

- For a certain input string, the output of a TM can be one in three kinds:
  - The TM **accepts** this string.
  - The TM **rejects** this string.
  - Rather than accepting or rejecting, the TM **loops** forever on the input string and never stops.



Very Important Terminology

- Let M be a Turing machine and let s be a string.
  - M **accepts** s if it enters an accepting state when running on s
  - M **rejects** s if it enters a rejecting state when running on s.
  - M **loops infinitely** on s when running on s if it enters neither an accepting nor a rejecting state.
  - M **does not accept** s if it either rejects s or loops on s.
  - M **does not reject** s if it either accepts s or loops on s.
  - M **halts** on s if it accepts w or rejects s.



More Details of RE

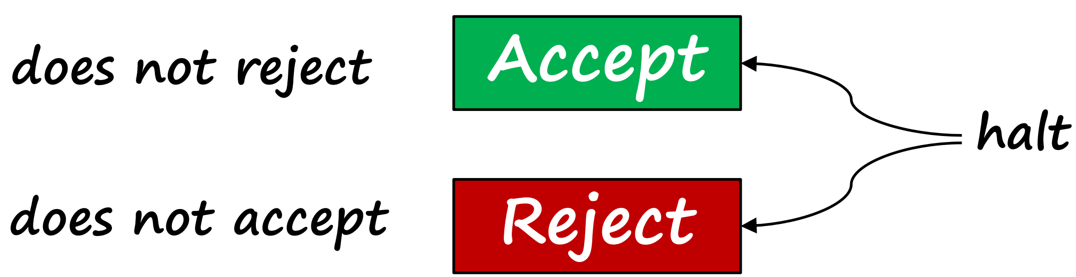
- We've known that for a certain language L, if there exists a TM $\boldsymbol{M}$ such that $L = L(\boldsymbol{M})$, then we call L is a **recursively enumerable language(递归可枚举语言)**，or ***RE***.
- So for any RE $\boldsymbol{L}$, a TM $\boldsymbol{M}$ that $L(M) = L$, and any string $\boldsymbol{w}$
  - If $w \in L, M$ accepts w in finite steps.
  - If $w \notin L, M$ does not accept w, it means M rejects w or loops forever
- We also call the TM $\boldsymbol{M}$ a **recognizer** for the language $\boldsymbol{L}$, and $\boldsymbol{L}$ is **Turing-recognizable(图灵可识别的).**

More Details of RE

- In other words, as an recognizer, $\boldsymbol{M}$ will tell you correct on every correct input, but $\boldsymbol{M}$ is not necessary to tell you wrong on every wrong input.
  - You can not determine the input is correct or wrong until $\boldsymbol{M}$ halts.
- But a problem is solvable(computable) if the computation process finishes in finite steps.

Decider

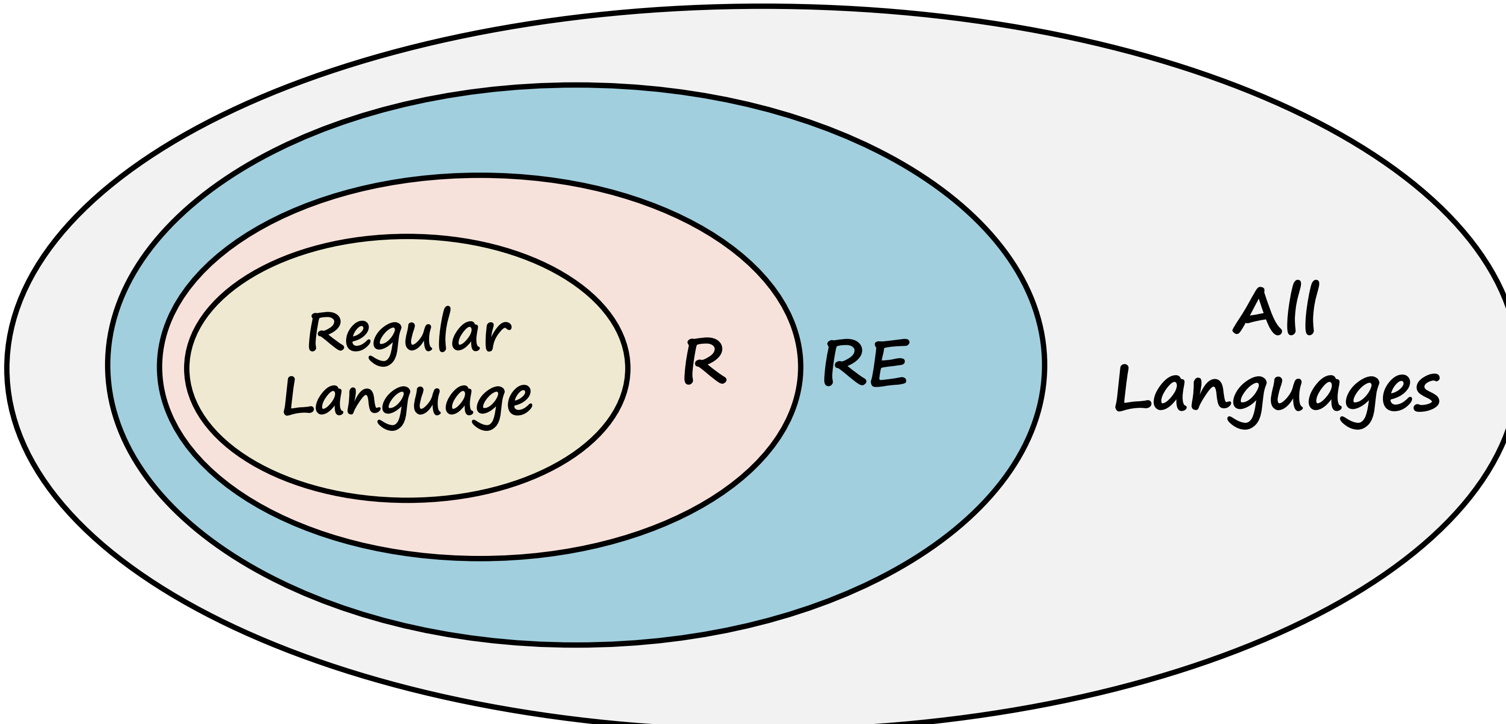
- Some Turing machines always halt; they never go into an infinite loop.
- If M is a TM and M halts on every possible input, then we say that M is a decider.
- For deciders, accepting is the same as not rejecting and rejecting is the same as not accepting.



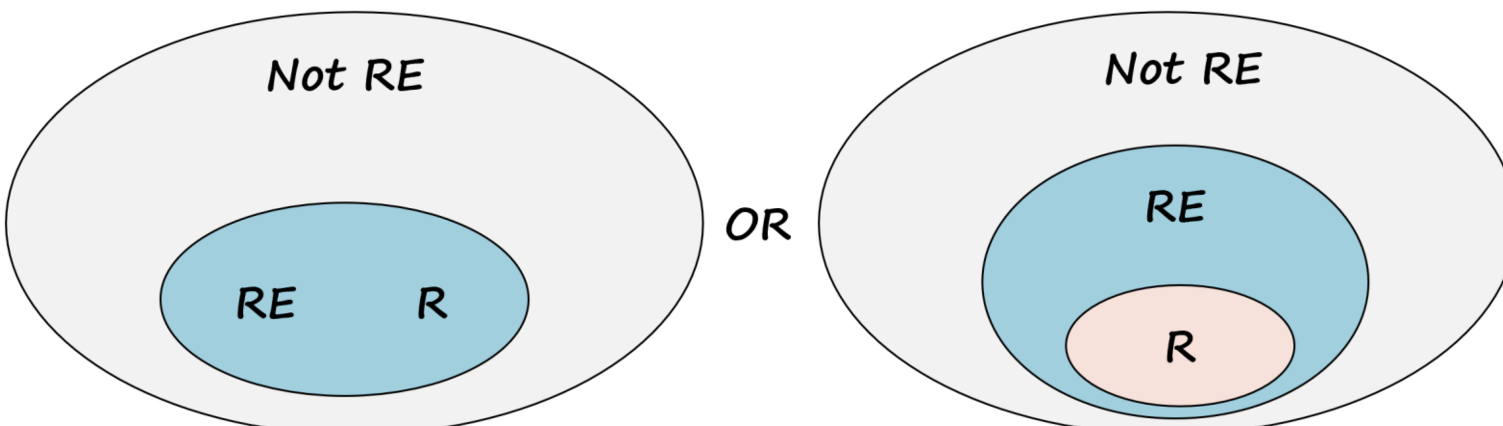
Recursive Language

- A language $\boldsymbol{L}$ is **recursive(递归的)** if there's a TM $\boldsymbol{M}$ such that:
  - If $w \in L, M$ accepts w in finite steps.
  - If $w \notin L, M$ rejects w in finite steps.
- We also call the TM $\boldsymbol{M}$ a **decider** for the language $\boldsymbol{L}$, and $\boldsymbol{L}$ is **Turing-decidable(图灵可判定的).**
- Decidable problems, in some sense, are that can definitely be "solved" by a computer.
- Recursive Language is a subset of RE language

Language Hierarchy



Question:
RE=R?



Next

Effective Computation on TM

- The universal TM.

Undecidability